

# Discrete and Continuous Dynamical Systems

Tutorial, 2018.02.14.

1. (a) Calculate the eigenvalues and eigenvectors of the following matrices!

$$\mathbf{G} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

- (b) Calculate the following quantities!

$$\det(\mathbf{H}), \det(\mathbf{G}), \operatorname{Tr} \mathbf{H}, \mathbf{G}^{-1}, \mathbf{H}^{-1}$$

2. (a) Calculate the following Laplace transforms!

$$\mathcal{L} \left\{ 3e^{2t} + \delta(t) + \frac{d4e^{-5t}}{dt} \right\} =$$

$$\mathcal{L} \left\{ \int_0^t e^{-3\tau} \eta(t - \tau) d\tau \right\} =$$

- (b) Calculate the following inverse Laplace transforms!

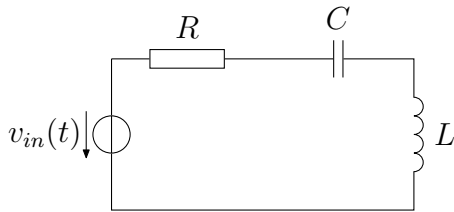
$$\mathcal{L}^{-1} \left\{ \frac{2}{s^2 + 3s} \right\} =$$

$$\mathcal{L}^{-1} \left\{ \frac{3s^2 + 12s + 11}{s^3 + 6s^2 + 11s + 6} \right\} =$$

3. Solve the following initial value problem using Laplace transform!

$$\ddot{y}(t) + \dot{y}(t) - 2y(t) = 4, \quad \dot{y}(0) = 1, \quad y(0) = 2$$

4. **Homework:** Given the following electrical network. The task is to determine inductors current for  $t \geq 0$ !



$$v_{in}(t) = \begin{cases} 0 \text{ V}, & t < 0 \\ 1 \text{ V}, & t \geq 0 \end{cases}$$

Matrix form:  $\mathbf{x}(t) = [ i_L(t) \quad v_C(t) ]^T$ ,  $y(t) = i_L(t)$ ,  $u(t) = v_{in}(t)$

- How many inputs and outputs does your system have?
- Which basic system properties hold for your system?
- From the basic equations of motion given, express your system in state space form!  
Substitute your parameter values ( $R, L, C$ ) into the obtained parametric model!

**Deadline of submission: 2018.02.21. 8am**

(Submit your homework in the moodle course in a hand written scanned pdf format!  
Please, write your name and neptun ID on the paper!)