Discrete and Continuous Dynamical Systems – tutorial
Continuous and discrete time nonlinear models

1 Lyapunov theorem for CT-LTI models

1. Given the following CT-LTI system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} -1 & 1 \end{bmatrix} x \end{aligned}$$

   - Give a possible Lyapunov function to this system and show that P fulfills the Lyapunov inequality. Give the corresponding Q.

2. Given the following CT-LTI system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} -1 & 1 \end{bmatrix} x \end{aligned}$$

   - Give a possible Lyapunov function to this system and show that P fulfills the Lyapunov inequality. Give the corresponding Q.

2 DT-LTI state space models and their automata model

1. Given the following DT-LTI system:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(k) \end{aligned} \tag{1}$$

Let us given the range set for the input $u(k) \in \{0, 1\}$ and for the states $x_i(k) \in \{-1, 0, 1\}$

- Give the state transition table of the corresponding automata model using identically zero input of your system.
- Extend the table with considering $u(k) = 1$.

3 Homework:

(a) Consider the following DT-LTI system

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.5 & -0.5 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(k) \end{aligned} \tag{2}$$

Let us given the range set for the input $u(k) \in \{0, 1\}$ and for the states $x_i(k) \in \{-1, -0.5, 0, 0.5, 1\}$ Compute the table of state transitions using identically zero input of your system.

(*) **(Supplementary)**
Extend the table with considering $u(k) = 1$.

Deadline of submission: 2018.04.11. 8am
(Submit your homework in the moodle course in a hand written scanned pdf format! Please, write your name and neptun ID on the paper!)