Digital Signal Processing
Signals and Systems

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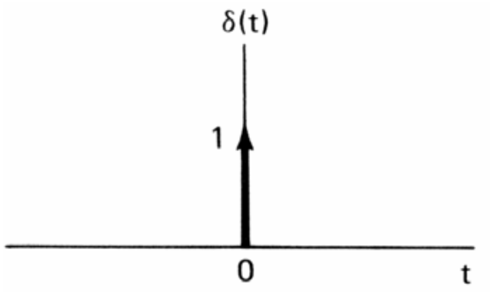
Special Signals – Dirac delta

- Reminder: signals are described as functions of time
- Dirac delta function:

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\mathrm{d}t = 1$$



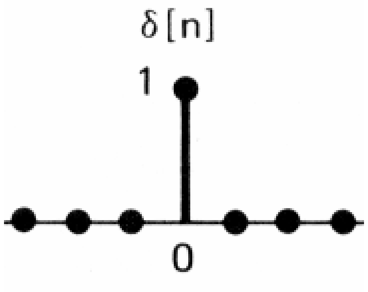
- Difficult to understand
- Infinitesimally narrow and infinitely tall at 0
- Represents the unit impulse in the analog case

Special Signals – Kronecker delta

- Kronecker delta (unit impulse) function:

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$\sum_{n=-\infty}^{\infty} \delta[n] = 1$$

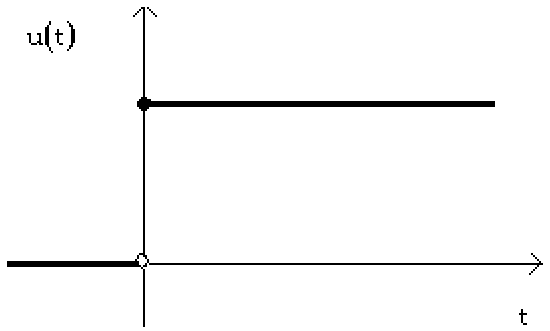


- Easy to understand
- Represents the unit impulse in the discrete case

Special Signals – Heaviside function

- Unit step function
- Analog case:

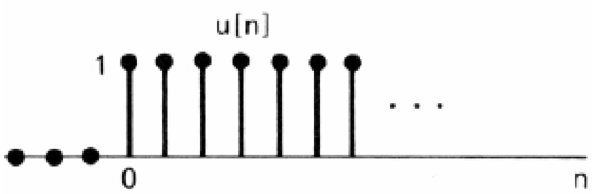
$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



  - Its value at 0 is a mystery…

- Discrete case:

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



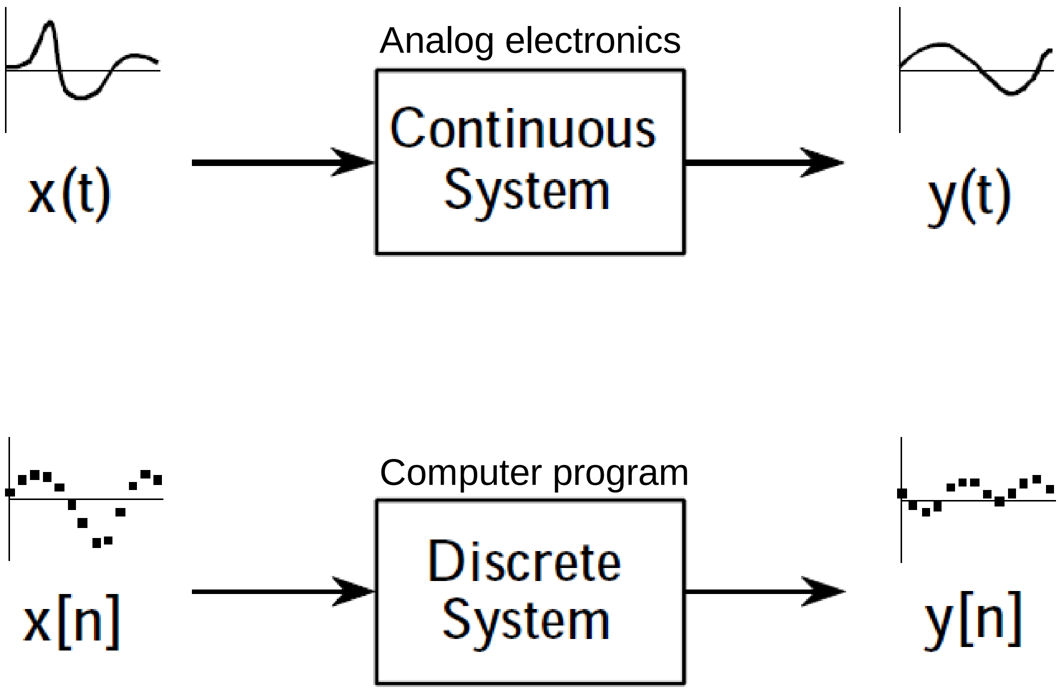
Definition(s) of System

- Any process that produces an output (signal) in response to an input (signal)
- Part of the world which interacts with the outside world by input(s) and output(s)
- Input, transformation and output
- Systems can be illustrated by block diagrams

Categories of System

- Many categorizations (similarly to signals), e.g.:
  - Analog: continous both in time and amplitude
  - Continous-time (CT): continous in time
  - Discrete-time (DT): discrete in time
  - Digital: discrete both in time and amplitude

Continous and Discrete System



System Properties

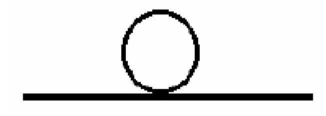
- Stability
  - Stable
  - Non-stable
- Linearity
  - Linear
  - Non-linear
- Behavior over time
  - Time-variant
  - Time-invariant
- Causality
  - Causal
  - Non-causal
  - Memoryless
- Number of inputs
  - SISO (Single Input Single Output)
  - MIMO (Multiple Input Multiple Output)
  - SIMO, MISO

Stability

- Local stable
  - If the input varies to a certain degree, the system returns to equilibrium after a while (with a finite distance from the original point)
- **BIBO** (bounded input bounded output
  - For any bounded input $|x(t)| < \infty$ the output of the system is also bounded $|y(t)| < \infty$
- Local asymptotically stable
  - If the input varies to a certain degree, the system returns to the original equilibrium point
- Global asymptotically stable
  - if the input varies to whatever degree, the system returns to the original equilibrium point

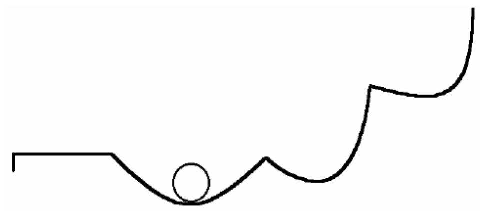
Stability – Examples

- Local stable
  - An orb on a table with a finite surface



- BIBO
  - An orb on a table with an infinite surface
- Local asymptotically stable
  - An orb in a finite bowl

- Global asymptotically stable
  - An orb in an infinite bowl
- A system with many equilibrium points:



Linearity and Time Invariance

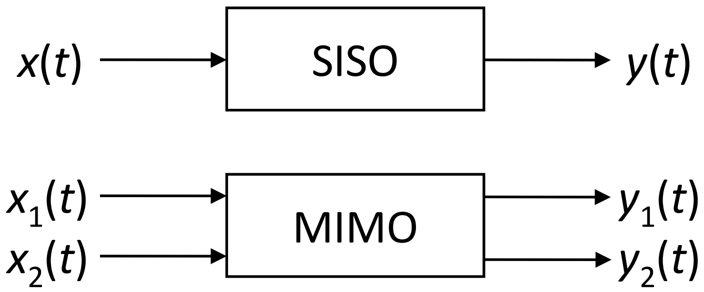
- Linearity
  - $x(t) = a_1 x_1(t) + a_2 x_2(t) + \cdots \leftrightarrow y(t) = a_1 y_1(t) + a_2 y_2(t) + \cdots$
  - Superposition
  - Divide and conquer strategy
- Time invariance
  - $x(t) = x_1(t - T) \rightarrow y(t) = y_1(t - T)$
  - If a delay appears in the input, the same delay will appear in the output

Causality

- Causal: the output depends only on the current or past values of the input, e.g.
  - $y[n] = x[n] + x[n-2]$
- Non-causal: the output depends also on future values of the input
  - $y[n] = x[n] - x[n+1]$
  - $y(t) = x\big(t - \sin(t)\big)$
- Memoryless: there is no time shift
  - $y[n] = 2x[n] - 4x[n]$

Number of Inputs

- SISO: input and output is a scalar
- MIMO: input and output is a vector



Example

- The accumulator:

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

  - Stable?
  - Linear?
  - Time invariant?
  - Causal?
  - Memoryless?
  - SISO?