Dynamic Modeling and Model Analysis of a Large Industrial Synchronous Generator

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***Abstract*— A simple dynamic model of an industrial size synchronous generator operating in a nuclear power plant is developed in this paper based on first engineering principles. The constructed state-space model consists of a nonlinear state equation and a bi-linear output equation. It has been shown that the model is locally asymptotically stable with parameters obtained from the literature for a similar generator.**

The effect of load disturbances on the partially controlled generator has been analyzed by simulation using a traditional PI controller. It has been found that the controlled system is stable and can follow the set-point changes in the effective power well. The disturbance rejection of the controller is also satisfactory.

I. Introduction

Nuclear power plants generate electrical power from nuclear energy, where the final stage of the power production includes a synchronous generator that is driven by a turbine. Similarly to other power plants, both the effective and reactive components of the generated power depend on the need of the consumers and on their own operability criteria. This consumer generated time-varying load is the major disturbance that should be taken care of by the generator controller.

The turbo generator, the subject of our study, is a specific synchronous generator with a special cooling system. The armature has been cooled by water and the rotor has been cooled by hydrogen. In the examined nuclear power station the exciter field regulator of the synchronous generator currently does not control the reactive power, only the effective power. *The final aim of our study is to design a controller that can control the reactive power such that its generation is minimized in such a way that the quality of the control of the effective power remains (nearly) unchanged.*

There are three generator exciter field regulators for each generators (automatic, manual, and the back-up). The manual generator exciter field regulator performs output voltage control of the synchronous generator by applying a sequential control to the output voltage of generator that is constrained by a voltage limiter. The sequential controller is a PI controller.

Because of the specialities and great practical importance of the synchronous generators in power plants, their modeling for control purposes is well investigated in the literature. Besides of the basic textbooks (see e.g. [1] and [2]), there are papers that describe the modeling and use the developed models for the design of various controllers [3], [4]. These papers, however, do not take the special circumstances found in nuclear power plants in to account and that may result in special generator models. The aim of this paper is to propose a simple dynamic model of a synchronous generator in a nuclear power plant for control studies together with a local stability analysis and model verification.

II. The model of the synchronous generator

In this section the state-space model for a synchronous generator is constructed that will be used for stability analysis and controller design based on [1] and [2].

A. The engineering model

For constructing the synchronous generator model, let us make the following assumptions:

- a symmetrical tri-phase stator winding system is assumed,
- one field winding is considered to be in the machine,
- there are two amortisseur or damper windings in the machine,
- all of the windings are magnetically coupled,
- the flux linkage of the winding is a function of the rotor position,
- the copper losses and the slots in the machine are neglected,
- the spatial distribution of the stator fluxes and apertures wave are considered to be sinusoidal,
- stator and rotor permeability are assumed to be infinite.

It is also assumed that all the losses due to wiring, saturation, and slots can be neglected.

The six windings (three stators, one rotor and two damper) are magnetically coupled. Since the magnetic coupling between the windings is a function of the rotor position, the flux linking of the windings is also a function of the rotor position. The actual terminal voltage v of the windings can be written in the form

$$v = \pm \sum_{j=1}^{J} (r_j \cdot i_j) \pm \sum_{j=1}^{J} (\dot{\lambda}_j),$$

where i_j are the currents, r_j are the winding resistances, and λ_j are the flux linkages. The positive directions of the stator currents point out of the synchronous generator terminals.

Thereafter, the two stator electromagnetic fields, both traveling at rotor speed, were identified by decomposing each stator phase current under steady state into two components, one in phase with the electromagnetic field and an other phase shifted by 90^o. With the aboves, one can construct an airgap field with its maxima aligned to the rotor poles (d axis), while the other is aligned to the q axis (between poles) (see Fig. 1).

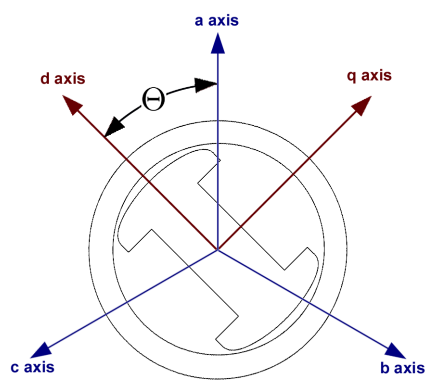

Fig. 1. The abc and $0dq$ frames of the generator

This method is called the Park's transformation that gives the following relationship:

$$\begin{aligned} \mathbf{i_{0dq}} &= \mathbf{P} \cdot \mathbf{i_{abc}} \\ \mathbf{i_{abc}} &= \mathbf{P^{-1}} \cdot \mathbf{i_{0dq}} \end{aligned} \tag{1}$$

where the current vectors are $\mathbf{i_{0dq}} = \begin{bmatrix} i_0 & i_d & i_q \end{bmatrix}^{\mathbf{T}}$ and $\mathbf{i_{abc}} = \begin{bmatrix} i_a & i_b & i_c \end{bmatrix}^{\mathbf{T}}$ and the Park's transformation matrix is:

$$\mathbf{P} = \sqrt{\frac{\mathbf{2}}{\mathbf{3}}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ i_a \cos(\Theta) & i_b \cos(\Theta - \frac{2\pi}{3}) & i_c \cos(\Theta - \frac{4\pi}{3}) \\ i_a \sin(\Theta) & i_b \sin(\Theta - \frac{2\pi}{3}) & i_c \sin(\Theta - \frac{4\pi}{3}) \end{bmatrix}$$

where i_a, i_b and i_c are the phase currents and Θ [rad] is the angle between the phase current i_a and the current i_d. Park's transformation uses three variables: d and q axis components (i_d and i_q) and stationary current component (i_0), which is proportional to the zero-sequence current.

All flux components correspond to an electromagnetic field (EMF), the generator EMF is primarily along the rotor q axis. The angle between this EMF and the output voltage is the machine torque angle δ, where the phase a is the reference voltage of the output voltage. The position of the d axis (in radian) is $\Theta = \omega_r t + \delta + \pi/2$, where ω_r is the rated synchronous angular frequency. Finally, the the voltage and linkage equations are $\mathbf{v_{0dq}} = \mathbf{P} \cdot \mathbf{v_{abc}}$ and $\lambda_{\mathbf{0dq}} = \mathbf{P} \cdot \lambda_{\mathbf{abc}}$, where the vectors are $\mathbf{v_{0dq}} = \begin{bmatrix} v_0 & v_d & v_q \end{bmatrix}^{\mathbf{T}}$ and $\mathbf{v_{abc}} = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix}^{\mathbf{T}}$, and the linkage flux vectors are $\lambda_{\mathbf{0dq}} = \begin{bmatrix} \lambda_0 & \lambda_d & \lambda_q \end{bmatrix}^{\mathbf{T}}$ and $\lambda_{\mathbf{abc}} = \begin{bmatrix} \lambda_a & \lambda_b & \lambda_c \end{bmatrix}^{\mathbf{T}}$.

The value of the active power can be written (using (1)) in both coordinate systems:

$$p = \mathbf{v_{abc}^T i_{abc}} = \mathbf{v_{0dq}^T P P^{-1} i_{0dq}} = \mathbf{v_{0dq}^T i_{0dq}} \tag{2}$$

B. The flux linkage equations

The generator consists of six coupled coils referred to with indices a, b, c (the stator phases coils), F, D, and Q (the field coil, the d-axis amortisseur and the q-axis amortisseur). The linkage equations are in the following form:

$$\begin{bmatrix} \lambda_a & \lambda_b & \lambda_c & \lambda_F & \lambda_D & \lambda_Q \end{bmatrix}^T =$$

$$\begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} \tag{3}$$

where L_{xy} is the coupling inductance of the coils. It is important to note that the inductances are time varying since Θ is a function of time. The time varying inductances can be simplified by referring all quantities to a rotor frame of reference through Park's transformation:

$$\begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \begin{bmatrix} \lambda_{\mathbf{abc}} \\ \lambda_{\mathbf{FDQ}} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} \mathbf{L_{aa}} & \mathbf{L_{aR}} \\ \mathbf{L_{Ra}} & \mathbf{L_{RR}} \end{bmatrix} \begin{bmatrix} \mathbf{P^{-1}} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix} \begin{bmatrix} \mathbf{i_{abc}} \\ \mathbf{i_{FDQ}} \end{bmatrix}, \tag{4}$$

where $\mathbf{L_{RR}}$ is the rotor-rotor, $\mathbf{L_{aa}}$ is the stator-stator, $\mathbf{L_{aR}}$ and $\mathbf{L_{Ra}}$ are the stator-rotor inductance matrices. $\mathbf{P}$ is the Park's transformation matrix, $\mathbf{I_3}$ is the 3×3 unit matrix. The obtained transformed flux linkage equations are as follows:

$$\begin{bmatrix} \lambda_0 & \lambda_d & \lambda_q & \lambda_F & \lambda_D & \lambda_Q \end{bmatrix}^T =$$

$$\begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix} \tag{5}$$

where:

$$\begin{array}{ll} L_d = L_s + M_s + \frac{3}{2} L_m & L_q = L_s + M_s - \frac{3}{2} L_m \\ L_0 = L_s - 2M_s & k = \sqrt{\frac{2}{3}} \end{array} \tag{6}$$

C. The voltage equations

The schematic equivalent circuit of the synchronous machine can be seen in Fig. 2, and the voltage equations (7) can be derived from it.

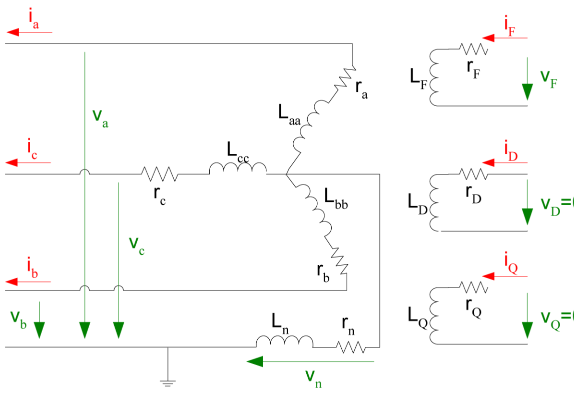

Fig. 2. The simplified schema of the synchronous machine

$$\begin{bmatrix} \mathbf{v_{abc}} \\ \mathbf{v_{FDQ}} \end{bmatrix} = -\begin{bmatrix} \mathbf{R_{abc}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix}\begin{bmatrix} \mathbf{i_{abc}} \\ \mathbf{i_{FDQ}} \end{bmatrix} - -\begin{bmatrix} \dot{\boldsymbol{\lambda}}_{\mathbf{abc}} \\ \dot{\boldsymbol{\lambda}}_{\mathbf{FDQ}} \end{bmatrix} + \begin{bmatrix} \mathbf{v_n} \\ 0 \end{bmatrix}, \tag{7}$$

where $\mathbf{R_{abc}} = \mathrm{diag}(\begin{bmatrix} r_a & r_b & r_c \end{bmatrix})$, and $\mathbf{R_{FDQ}} = \mathrm{diag}(\begin{bmatrix} r_F & r_D & r_Q \end{bmatrix})$.

The neutral voltage $\mathbf{v_n}$ can also be deduced from Fig. 2 as follows:

$$\mathbf{v_n} = -\mathbf{R_n}\mathbf{i_{abc}} - \mathbf{L_{nm}}\dot{\mathbf{i}}_{\mathbf{abc}}, \tag{8}$$

where $\mathbf{L_{nm}} = L_n\mathbf{U_3}$, and $\mathbf{R_n} = r_n\mathbf{U_3}$, and $\mathbf{U_3}$ denotes the 3×3 matrix of full ones.

The direct, quadratic, field and amortisseur component of the voltage using Park's transformation:

$$\begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix}\begin{bmatrix} \mathbf{v_{abc}} \\ \mathbf{v_{FDQ}} \end{bmatrix} = \begin{bmatrix} \mathbf{v_{0dq}} \\ \mathbf{v_{FDQ}} \end{bmatrix} \tag{9}$$

Using (1), it is possible to expand the voltages of the resistances from (9) as

$$\begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix}\begin{bmatrix} \mathbf{R_{abc}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix}\begin{bmatrix} \mathbf{i_{abc}} \\ \mathbf{i_{FDQ}} \end{bmatrix} =$$
$$= \begin{bmatrix} \mathbf{P R_{abc} P^{-1}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix}\begin{bmatrix} \mathbf{i_{0dq}} \\ \mathbf{i_{FDQ}} \end{bmatrix} =$$
$$= \begin{bmatrix} \hat{\mathbf{R}}_{\mathbf{abc}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix}\begin{bmatrix} \mathbf{i_{0dq}} \\ \mathbf{i_{FDQ}} \end{bmatrix}.$$

Using the initial assumption of symmetrical tri-phase stator windings (i.e. $r_a = r_b = r_c = r$), we obtain $\hat{\mathbf{R}}_{\mathbf{abc}} = \mathbf{R_{abc}} = \mathrm{diag}(\begin{bmatrix} r & r & r \end{bmatrix})$.

The time derivatives of the fluxes can be computed similarly

$$\begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I_3} \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{\lambda}}_{\mathbf{abc}} \\ \dot{\boldsymbol{\lambda}}_{\mathbf{FDQ}} \end{bmatrix} = \begin{bmatrix} \mathbf{P}\dot{\boldsymbol{\lambda}}_{\mathbf{abc}} \\ \dot{\boldsymbol{\lambda}}_{\mathbf{FDQ}} \end{bmatrix}, \tag{10}$$

where

$$\mathbf{P}\dot{\boldsymbol{\lambda}}_{\mathbf{abc}} = \dot{\boldsymbol{\lambda}}_{\mathbf{0dq}} - \dot{\mathbf{P}}\boldsymbol{\lambda}_{\mathbf{abc}} = \dot{\boldsymbol{\lambda}}_{\mathbf{0dq}} - \dot{\mathbf{P}}\mathbf{P^{-1}}\boldsymbol{\lambda}_{\mathbf{0dq}},$$

and the last term is

$$\dot{\mathbf{P}}\mathbf{P^{-1}}\boldsymbol{\lambda}_{\mathbf{0dq}} = \omega\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \end{bmatrix} = \begin{bmatrix} 0 \\ -\omega\lambda_q \\ \omega\lambda_d \end{bmatrix}$$

Finally, the neutral voltage is derived as

$$\begin{bmatrix} \mathbf{v_{0dq}} \\ \mathbf{v_{FDQ}} \end{bmatrix} = -\begin{bmatrix} \mathbf{R_{0dq}} & 0 \\ 0 & \mathbf{R_{FDQ}} \end{bmatrix}\begin{bmatrix} \mathbf{i_{0dq}} \\ \mathbf{i_{FDQ}} \end{bmatrix}$$
$$-\begin{bmatrix} \dot{\boldsymbol{\lambda}}_{\mathbf{0dq}} \\ \dot{\boldsymbol{\lambda}}_{\mathbf{FDQ}} \end{bmatrix} + \begin{bmatrix} \dot{\mathbf{P}}\mathbf{P^{-1}}\boldsymbol{\lambda}_{\mathbf{0dq}} \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{n_{0dq}} \\ 0 \end{bmatrix} \tag{11}$$

where $\mathbf{n_{0dq}}$ is the voltage drop from the neutral network.

$$\mathbf{n_{0dq}} = \mathbf{P v_n} = -\mathbf{P R_n P^{-1} P i_{abc}} - \mathbf{P L_{nm} P^{-1} P}\dot{\mathbf{i}}_{\mathbf{abc}} =$$
$$-\mathbf{P R_n P^{-1} i_{0dq}} - \mathbf{P L_{nm} P^{-1}}\dot{\mathbf{i}}_{\mathbf{0dq}} = \begin{bmatrix} -3r_n i_0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -3L_n\dot{i}_0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

In balanced condition the v_0 voltage is 0. The above equation can be written in the following form:

$$\begin{bmatrix} \mathbf{v_{dq}} & \mathbf{v_{FDQ}} \end{bmatrix}^T =$$
$$-\begin{bmatrix} R & 0 \\ 0 & R_R \end{bmatrix}\begin{bmatrix} \mathbf{i_{dq}} \\ \mathbf{i_{FDQ}} \end{bmatrix} - \begin{bmatrix} \dot{\boldsymbol{\lambda}}_{\mathbf{dq}} \\ \dot{\boldsymbol{\lambda}}_{\mathbf{FDQ}} \end{bmatrix} + \begin{bmatrix} S \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{n_{0dq}} \\ 0 \end{bmatrix} \tag{13}$$

where $\mathbf{R} = \mathrm{diag}(\begin{bmatrix} r & r \end{bmatrix})$, $\mathbf{R_R} = \mathrm{diag}(\begin{bmatrix} r_F & r_D & r_Q \end{bmatrix})$, and $\mathbf{S} = \begin{bmatrix} -\omega\lambda_q & \omega\lambda_d \end{bmatrix}^{\mathbf{T}}$

We can write the voltage equation in simplified matrix form as

$$\mathbf{v_{dFDqQ}} = -\mathbf{R_{RS\omega}}\mathbf{i_{dFDqQ}} - \mathbf{L}\dot{\mathbf{i}}_{\mathbf{dFDqQ}}, \tag{14}$$

where $\mathbf{v_{dFDqQ}} = \begin{bmatrix} v_d & -v_F & v_D = 0 & v_q & v_Q = 0 \end{bmatrix}^{\mathbf{T}}$, $\mathbf{i_{dFDqQ}} = \begin{bmatrix} i_d & i_F & i_D & i_q & i_Q \end{bmatrix}^{\mathbf{T}}$ while $\mathbf{R_{RS\omega}}$ and $\mathbf{L}$ are the following expressions

$$\mathbf{R_{RS\omega}} = \begin{bmatrix} r & 0 & 0 & \omega L_q & \omega k M_Q \\ 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 \\ -\omega L_d & -\omega k M_F & -\omega k M_D & r & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} L_d & kM_F & kM_D & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & L_q & kM_Q \\ 0 & 0 & 0 & kM_Q & L_Q \end{bmatrix}$$

The state-space model for the currents is obtained by expressing $\dot{\mathbf{i}}_{\mathbf{dFDqQ}}$ from (14), i.e.

$$\dot{\mathbf{i}}_{\mathbf{dFDqQ}} = -\mathbf{L}^{-1} \cdot \mathbf{R_{RS\omega}} \cdot \mathbf{i_{dFDqQ}} - \mathbf{L}^{-1} \cdot \mathbf{v_{dFDqQ}} \tag{15}$$

D. Torque

The next step is to derive the mechanical part of the model [2]. The energy balance is written in the form

$$dW_{out} = dW_{Mech} - dW_{Field} + dW_{\Omega}, \tag{16}$$

where W_{Ω} is the energy losses in the resistance of the machine, W_{Field} is the energy of the field, W_{Mech} is the mechanical energy and W_{out} is the output energy of the synchronous generator. The time derivative of (16) is the power equation:

$$\frac{dW_{out}}{dt} = \frac{dW_{Mech}}{dt} - \frac{dW_{Field}}{dt} - \frac{dW_{\Omega}}{dt} \tag{17}$$

$$p_{out} = p_{Mech} - p_{Field} - p_{\Omega} \tag{18}$$

On the other hand, using (2), the output power of tri-phase system is:

$$p_{out} = \mathbf{v_{abc}^T i_{abc}} = \mathbf{v_{0dq}^T i_{0dq}} \tag{19}$$

The mechanical torque (T_{Mech}) is obtained by dividing power by the angular velocity $\omega = \frac{d\theta}{dt}$, i.e. $T_{Mech} = \frac{P_{Mech}}{\omega}$. This gives

$$T_{Mech} = \lambda_d i_q - \lambda_q i_d \tag{20}$$

The torque equation which gives the torque of the field energy is:

$$T_{Field} = (i_0 \frac{\lambda_0}{d\theta} + i_d \frac{\lambda_d}{d\theta} + i_q \frac{\lambda_q}{d\theta}) \tag{21}$$

On the other hand, the electrical torque can be expressed as

$$T_{Electr} = T_{Mech} - T_{Field} \tag{22}$$

From Newton's second law the equation of motion is

$$\frac{2H}{\omega_B} \dot{\omega} = T_{Mech} - T_{Electr} - T_{Dump}, \tag{23}$$

where H is the inertia constant and T_{Dump} is the dumping torque. The time and the rotation speed using per units i.e. dimensionless variables is $t_u = \omega_B t$ and $\omega_u = \omega / \omega_B$. Afterwards the normalized swing equation can be written as

$$2H\omega_B \frac{d\omega_u}{dt_u} = T_{Acc}, \tag{24}$$

where $2H\omega_B = \tau_j$, where τ_j is a time-like quantity coming from per unit notation not detailed here.

The electrical torque can be expressed from the flux and the current of the machine

$$T_{Electr} = \frac{1}{3}(\lambda_d i_q - \lambda_q i_d) \tag{25}$$

The total torque accelerating the generator is then

$$T_{Acc} = T_{Mech} - \frac{T_{Elect3}}{3} - T_{Dump} = T_{Mech} - T_{Electr} - T_{Dump},$$

where T_{Mech} is the mechanical torque, T_{Electr} is the electrical torque per phase, T_{Elect3} is the electrical torque for all 3 phases. It is often convenient to write the damping torque as $T_{Dump} = D\omega$, where D is a damping constant.

The electrical torque expressed using the state vector of model (15) is then

$$T_{Elect3} = \begin{bmatrix} L_d i_q \\ kM_F i_q \\ kM_D i_q \\ -L_q i_d \\ -kM_Q i_d \end{bmatrix}^T \cdot \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \end{bmatrix}. \tag{26}$$

Since the variables have a few orders of magnitude difference in their values in natural units, the equations are normalized with respect to a base value (corresponding to the normal range of the variables). This way all signals are measured in normalized units (p.u.). Since $\dot{\omega} = \frac{T_{Acc}}{\tau_j}$, the speed of the synchronous machine is

$$\dot{\omega} = \begin{bmatrix} -\frac{L_d i_q}{3\tau_j} \\ -\frac{kM_F i_q}{3\tau_j} \\ -\frac{kM_D i_q}{3\tau_j} \\ \frac{L_q i_d}{3\tau_j} \\ \frac{kM_Q i_d}{3\tau_j} \\ -\frac{D}{\tau_j} \end{bmatrix}^T \cdot \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \\ \omega \end{bmatrix} + \frac{T_{Mech}}{\tau_j} \tag{27}$$

Note, that (27) can be used as a supplementary state equation for state space model (15). The loading angle (δ) of the synchronous generator is

$$\delta = \delta_0 + \int_{t_0}^{t} (\omega - \omega_r) dt$$

that we can differentiate to obtain the time derivative of the δ in per unit notation

$$\dot{\delta} = \omega - 1, \tag{28}$$

so the loading angle (δ) can also be regarded as a state variable in the state space model (15, 27). Altogether, there are 6 state variables: i_d , i_F , i_D , i_q , i_Q , ω and δ. The input variables (i.e. manipulatable inputs and disturbances) are: T_{Mech}, v_F, v_d and v_q. Observe, that the state equations (15, 27, 28) *are bilinear in the state variables* because matrix $\mathbf{R_{RS\omega}}$ in (15) depends linearly on ω.

E. The output equation of the model

The output active power equation can be written in the following form:

$$p_{out} = v_d i_d + v_q i_q + v_0 i_0 \tag{29}$$

Assuming steady-state for the stationary components ($v_0 = i_0 = 0$), (29) simplifies to

$$p_{out} = v_d i_d + v_q i_q, \tag{30}$$

and the reactive power is

$$q_{out} = v_d i_q - v_q i_d. \tag{31}$$

Equations (30-31) are the *output equations* of the generator's state-space model. Observe, that these equations *are bi-linear in the state and input variables.*

F. Connecting the synchronous generator to an infinite huge network

Since every synchronous machine is connected to an infinite bus (shown in Fig. 3.) the next task is to extend the previous models with an infinite bus. In Fig. 3, resistance R_e and inductance L_e represent the output transformer of the synchronous generator and the transmission-line.

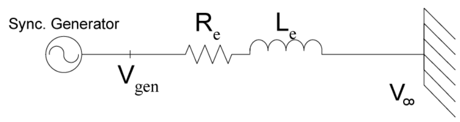

Fig. 3. Synchronous machine connected to an infinite bus

The matrix form of the modified voltage equation is as follows:

$$\mathbf{v_{abc}} = \mathbf{v_{\infty abc}} + R_e \mathbf{I_3} \mathbf{i_{abc}} + L_e \mathbf{I_3} \dot{\mathbf{i}}_{\mathbf{abc}} \tag{32}$$

Equation (32) can be transformed to the $0dq$ coordinate system as

$$\mathbf{v_{0dq}} = \mathbf{P v_{abc}} = \mathbf{P v_{\infty abc}} + R_e \mathbf{I_3} \mathbf{i_{0dq}} + L_e \mathbf{I_3} \dot{\mathbf{i}}_{\mathbf{0dq}} \tag{33}$$

The tri-phase voltage of the bus in the $0dq$ coordinate system is then

$$\mathbf{v_{\infty 0dq}} = \mathbf{P v_{\infty abc}} = \sqrt{\mathbf{3}} \mathbf{V_\infty} \begin{bmatrix} 0 \\ -\sin(\delta - \alpha) \\ \cos(\delta - \alpha) \end{bmatrix} \tag{34}$$

Afterwards, one can express the current vector $\mathbf{i_{0dq}}$ and voltage vector $\mathbf{v_{0dq}}$ as

$$\mathbf{P} \dot{\mathbf{i}}_{\mathbf{abc}} = \dot{\mathbf{i}}_{\mathbf{0dq}} - \dot{\mathbf{P}} \mathbf{i_{abc}} = \dot{\mathbf{i}}_{\mathbf{0dq}} - \dot{\mathbf{P}} \mathbf{P^{-1}} \mathbf{i_{0dq}} \tag{35}$$

and

$$\mathbf{v_{0dq}} = \mathbf{v_\infty} \sqrt{\mathbf{3}} \begin{bmatrix} 0 \\ -\sin(\delta - \alpha) \\ \cos(\delta - \alpha) \end{bmatrix} + \mathbf{R_e} \mathbf{i_{0dq}} + \mathbf{L_e} \dot{\mathbf{i}}_{\mathbf{0dq}} - \omega \mathbf{L_e} \begin{bmatrix} 0 \\ -i_q \\ i_d \end{bmatrix} \tag{36}$$

The integration of resistance R_e and inductance L_e into voltage equation (14) can be done by a simple change in matrices $\mathbf{R}_{\mathbf{RS}\omega}$ and $\mathbf{L}$.

The obtained voltage equation is:

$$\mathbf{v}_{\mathbf{dFDqQ}} = \tilde{\mathbf{R}}_{\mathbf{RS}\omega}\mathbf{i}_{\mathbf{dFDqQ}} + \tilde{\mathbf{L}}\dot{\mathbf{i}}_{\mathbf{dFDqQ}}, \tag{37}$$

where $\mathbf{v}_{\mathbf{dFDqQ}}$, $\mathbf{i}_{\mathbf{dFDqQ}}$, $\dot{\mathbf{i}}_{\mathbf{dFDqQ}}$ $\mathbf{R}_{\mathbf{RS}\omega}$ and $\tilde{\mathbf{L}}$ are

$$\begin{aligned}
\mathbf{v}_{\mathbf{dFDqQ}} &= \begin{bmatrix} v_d & -v_F & v_D = 0 & v_q & v_Q = 0 \end{bmatrix}^T \\
\mathbf{i}_{\mathbf{dFDqQ}} &= \begin{bmatrix} i_d & i_F & i_D & i_q & i_Q \end{bmatrix}^T \\
\tilde{\mathbf{R}}_{\mathbf{RS}\omega} &= \mathbf{R}_{\mathbf{RS}\omega} + \mathrm{diag}(\begin{bmatrix} R_e & 0 & 0 & R_e & 0 \end{bmatrix}) \\
\tilde{\mathbf{L}} &= \mathbf{L} + \mathrm{diag}(\begin{bmatrix} L_e & 0 & 0 & L_e & 0 \end{bmatrix})
\end{aligned}$$

From (37) it is possible to express $\dot{\mathbf{i}}_{\mathbf{dFDqQ}}$ as

$$\dot{\mathbf{i}}_{\mathbf{dFDqQ}} = -\tilde{\mathbf{L}}^{-1}\tilde{\mathbf{R}}_{\mathbf{RS}\omega}\mathbf{i}_{\mathbf{dFDqQ}} - \tilde{\mathbf{L}}^{-1}\mathbf{v}_{\mathbf{dFDqQ}} \tag{38}$$

III. Simulation Results

The above model has been verified by simulation against engineering intuition using parameter values of a similar generator taken from the literature.

A. Generator parameters

The parameters of the synchronous generator were obtained from the literature [1]:

Apparent energy	160 MVA,
$\cos\varphi$	0.85,
Voltage	15 kV,
Current	6158 A,
Frequency	60 Hz,
Excitation current	926 A,
Excitation voltage	375 V.

The stator base quantities, the rated power, output voltage, output current and the angular frequency are:

$$\begin{aligned}
S_B &= 160\text{ MVA}/3 = 53.333\text{ MVA} \\
V_B &= 15\text{ kV}/\sqrt{3} = 8.66\text{ kV} \\
I_B &= 6158\text{ A} \\
\omega_e &= 2\pi f\text{ rad/s}
\end{aligned}$$

Finally, the parameters of the synchronous machine and the external network in per units are:

$L_d = 1.700$	$l_d = 0.150$	$L_{MD} = 0.02838$
$L_q = 1.640$	$l_q = 0.150$	$L_{MQ} = 0.2836$
$L_D = 1.605$	$l_F = 0.101$	$r = 0.001096$
$L_Q = 1.526$	$l_D = -0.055$	$r_F = 0.00074$
$L_{AD} = 1.550$	$l_Q = 0.036$	$r_D = 0.0131$
$L_{AQ} = 1.490$	$r_Q = 0.054$	$R_e = 0.2$
$V_\infty = 0.828$	$L_e = 1.640$	$D = 2.004$

The steady-state values of the state variables can be obtained from the steady-state version of state equations (15, 27, 28) using the above parameters. Equation (14) implies that the expected value to i_D and i_Q are 0, that coincide with the engineering intuition. The equilibrium point of the system is:

$\omega = 0.9990691$, $i_d = -1.9132609$, $i_q = 0.66750001$, $i_F = 2.97899982$, $i_D = -8.6242856 \cdot 10^{-9}$, $i_Q = -5.3334899 \cdot 10^{-10}$

The state matrix $\mathbf{A}$ of the locally linearized state-space model $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ has the following numerical value:

$$\begin{bmatrix}
-0.0361 & 0.0004 & 0.0142 & -3.4851 & -2.5455 & -2.3285 \\
0.0124 & -0.0049 & 0.0772 & 1.2011 & 0.8773 & 0.8025 \\
0.0228 & 0.0044 & -0.0964 & 2.2057 & 1.6110 & 1.4737 \\
3.5855 & 2.6464 & 2.6464 & -0.0361 & 0.0901 & 1.0247 \\
-3.5009 & -2.5839 & -2.5839 & 0.0352 & -0.1234 & -1.0005 \\
-8 \cdot 10^{-6} & -0.0002 & -0.0002 & -0.0008 & -0.0005 & -0.0011
\end{bmatrix}$$

The eigenvalues of the state matrix are:

$$\begin{aligned}
&\lambda_{1,2} = -3.619088 \cdot 10^{-2} \pm && j0.997704 \\
&\lambda_3 = -0.100024 && \lambda_4 = -1.67235 \cdot 10^{-3} \\
&\lambda_5 = -4.724291e \cdot 10^{-4} && \lambda_6 = -0.123426
\end{aligned}$$

It is apparent that the real part of the eigenvalues are negative but their magnitudes are small, thus the system is on the boundary of the stability domain. Furthermore, $\lambda_{1,2}$ are complex conjugated pair with a relatively large imaginary part, that indicates the presence of an oscillatory component in the response. This behavior was expected since the direct and the quadrature equivalent circuit of the synchronous generator consist only of resistances and inductances, where the resistances give the windings and the iron losses, which are designed to be small in order to decrease the heating of the generator.

B. Validation of the model

The dynamic properties of the generator have been investigated in such a way that a single synchronous machine was connected to an infinite bus that models the electrical network (see Fig. 3). First, the response of the speed controlled generator has been tested under step-like changes of the exciter voltage. The simulation results are shown in Fig. 4, where the exciter voltage v_F and the torque angle δ are shown.

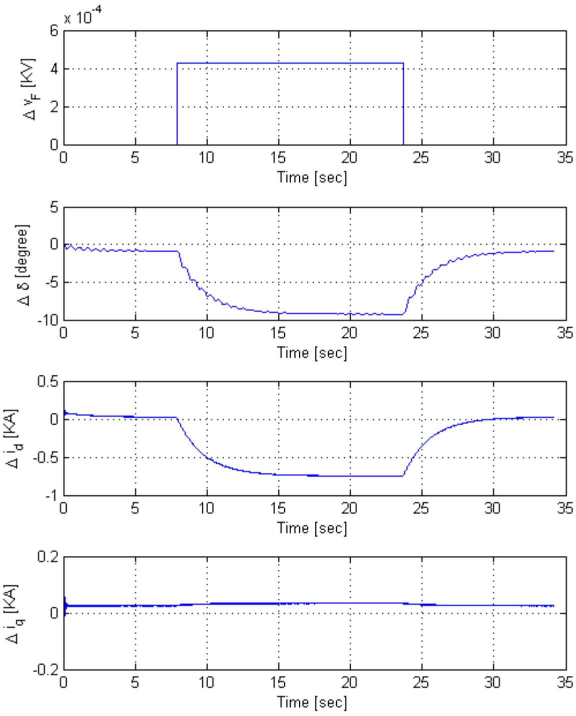

Fig. 4. Response to the exciter voltage step change of the controlled generator (Δ means the deviation form the steady-state value)

C. Changing the effective power of the generator

The control schemes of synchronous machines are commonly based on a reduced-order linearized model and a classical PI controller that ensures stability of the equilibrium point under small perturbations [4]. The controlled outputs are the effective power (p_{out}) and the speed (ω), the manipulated input are the exciter voltage (v_F) and the mechanical torque T_{Mech}. The proportional parameter of the PI controller of the speed is 0.5 and the integrator time is 0.5 in per units, the parameters of the effective power controller are P=0.02 and I=0.02. The response of the controlled generator has been tested under step-like changes in the setpoint of the effective power. The simulation results can be seen in Fig. 5, where the currents and the power components (effective and reactive) are shown. It is apparent that both the effective and the

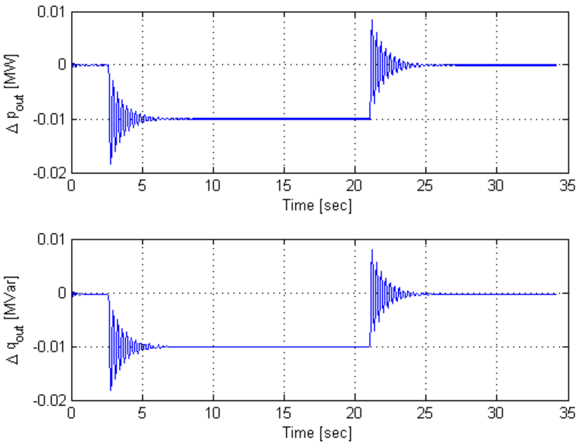

Fig. 5. Changing the effective power of the generator

reactive power follow the setpoint changes well, and the controlled system is fairly stable.

D. The effect of disturbances from the network

The effect of the disturbances from the electrical network is modeled by using a noise from the infinite bus that appears in the voltage variable v_q. The same step-like changes in the setpoint of the effective power has been applied as before.

Fig. 6. shows the simulation results. Here again, both the effective and the reactive power follow the setpoint changes well, and the controlled system is stable. In addition, the controller rejects the disturbances well.

IV. Conclusion and further work

A simple dynamic model of an industrial size synchronous generator operating in a nuclear power plant is developed in this paper based on first engineering principles. The constructed state-space model consists of a bilinear state equation originating from the flux linkage equations, and a bilinear output equation giving the effective and reactive power of the generator.

It has been shown that the model is locally asymptotically stable around a physically meaningful equilibrium state with parameters obtained from the literature for a similar generator.

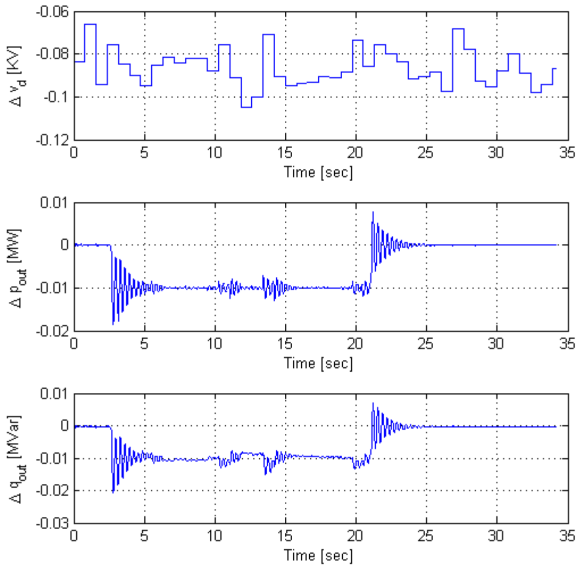

Fig. 6. The effect of the network disturbances

The effect of load disturbances on the partially controlled generator has been analyzed by simulation by using a traditional PI controller. It has been found that the controlled system is stable and can follow the setpoint changes in the effective power well. The disturbance rejection of the controller is also satisfactory.

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