Dynamic system modeling for control and diagnosis

Modelling of mechanical systems

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Mechanical systems - 1

Characterization: through mechanisms and modeling objects

Modeling objects: called "bodies" for which conservation balances are constructed

- elementary object: ***mass point***
- models of "mass point systems" result in concentrated parameter models
- rigid bodies
- deformable bodies

Mechanisms: for which constitutive equations are available

- forces originating from a potential field, e.g. *spring*
- gravitation
- friction
- torsion, deformation (elastic, non-elastic), etc.

Mechanical systems - 2

Basic principles: Newton - Principia Mathematica Phylosophiae Naturalis (1687)

1. Each object (body) keeps its steady-state position or its rectilinear motion with constant velocity unless an external effect forces it to change.
   - equivalent coordinate systems - move with constant relative velocity
   - external effects - described by ***forces***
2. The change of ***momentum*** is proportional to the effect of the force and takes place in the direction of the force.
   - both the momentum and the force are vectors, thus they have magnitude and direction
   - ***momentum is a conserved extensive quantity***
3. The effect is equal to the reaction, that is, the effect of two bodies to each other is of equal magnitude but of adverse direction.

Rectilinear motions

Modeling of mechanical systems - 1

I. Rectilinear motions
simplest case: mass-point systems

Conservation balances: momentum balance in each considered direction for each mass point

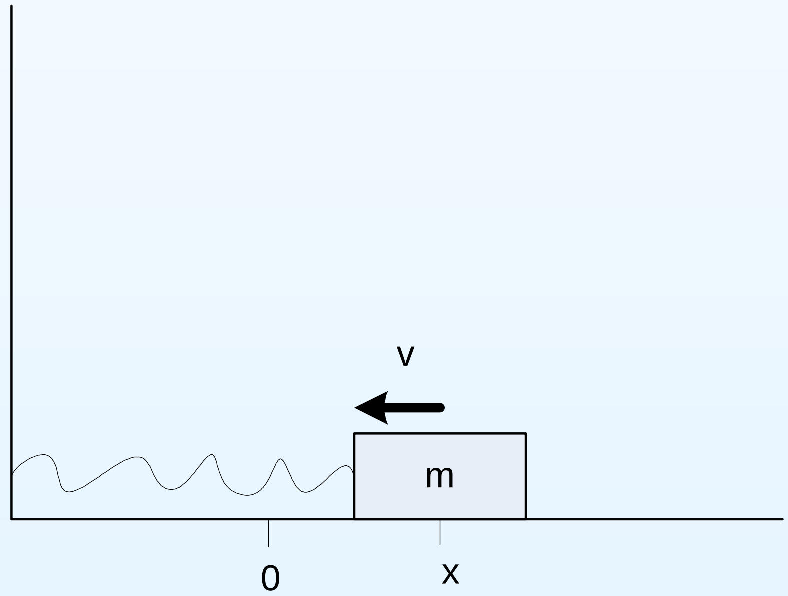
Constitutive equations

- *static equation(s) for each force* from the physics of its underlying mechanism
  - gravitation: $F_g = m \cdot g$
  - friction: $F_f = \mu \cdot m \cdot g$
  - elastic force (e.g. in spring): $F_s = -k \cdot x$ ***position dependent!***
  - other position dependent forces (e.g. from a potential, electrical forces)
- *dynamic position-momentum equation* (if needed)

Example: mass and spring

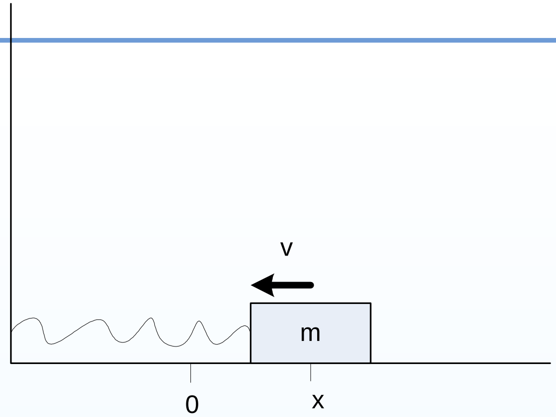
Problem description

Given a mass point m connected to the wall by an elastic spring with spring coefficient k and moving along a horizontal line. The mass is under the influence of an external force F_e and there is a friction with a coefficient μ.



Construct the model of the mass-spring system for control purposes if we can measure the position of the mass point x and the external force F_e.

Example: mass and spring - 2



Mechanisms

- spring
- friction generated by the gravitational force
- external force F_e

Modeling assumptions

F1 one mass point with mass m

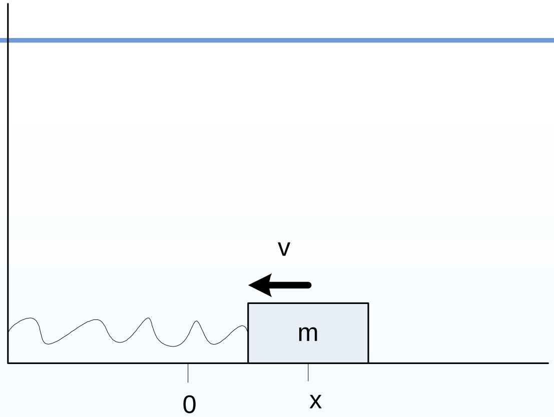
F2 rectilinear motion

F3 elastic ideal spring with coefficient k (force F_s)

F4 friction with constant coefficient μ (force F_f)

F5 position origo $x = 0$ is at the equilibrium point

Example: mass and spring - 3



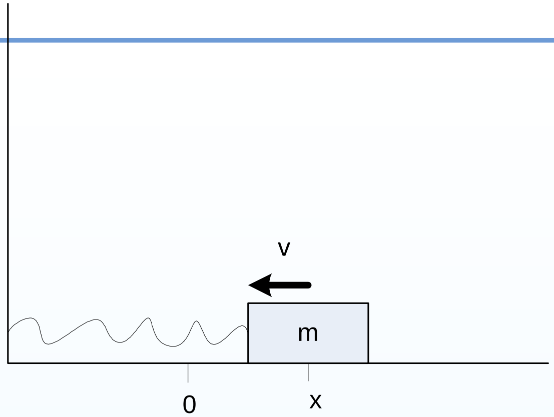
Conservation balance equation:
for momentum $p = m \cdot v$

$$\frac{dp}{dt} = F_f + F_s + F_e \qquad (1)$$

Constitutive equations

- $F_f = \mu \cdot m \cdot g$ (friction)
- $F_s = -k \cdot x$ (spring)
- $p = m \cdot v = m \cdot \frac{dx}{dt}$ (position is measurable)

Example: mass and spring - 4



Model equations with measurable variables:

$$\frac{dp}{dt} = \mu \cdot m \cdot g - k \cdot x + F_e \qquad (2)$$

$$\frac{dx}{dt} = \frac{1}{m} p \qquad (3)$$

State-space model form

- state variables: position x and momentum (p, or v)
- input variable: external force F_e
- output variable: position x

Rotating motions

Modeling of mechanical systems - 2

II. Rotating motions
simplest case: mass-point systems

Characteristic variables and parameters

- angular momentum N (analogue to the momentum p)
- angle velocity (angular frequency) ω ($[\frac{rad}{s}]$)
- moment of inertia Θ (for mass point m_i: $\Theta_i = m_i \cdot r_i^2$
- torque $M = F \times r$ in $[Nm]$ - r is the distance from the rotating axis

Conservation balances: angular momentum balance in each considered direction for each rotating mass point

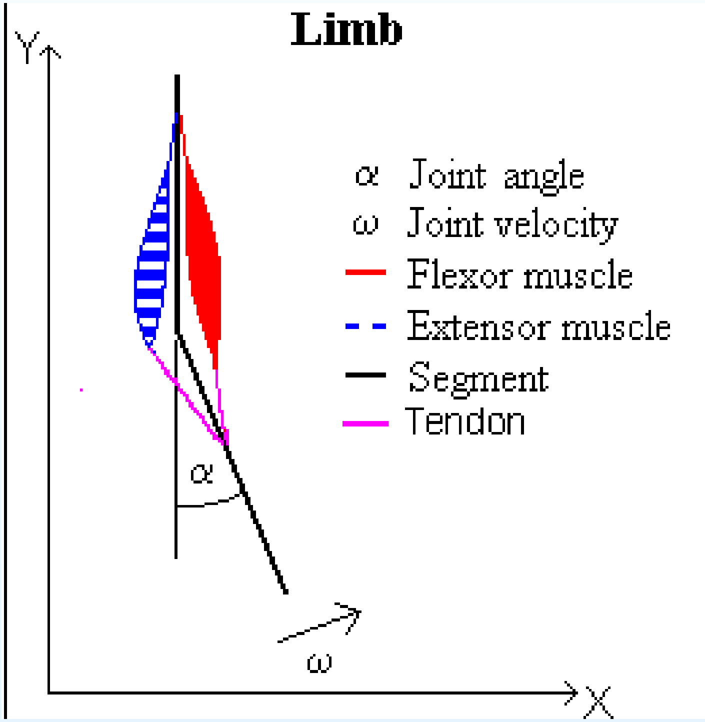
$$\frac{dN}{dt} = \sum_k M_k \tag{4}$$

with the external torques M_k

Example: a simple limb model - 1

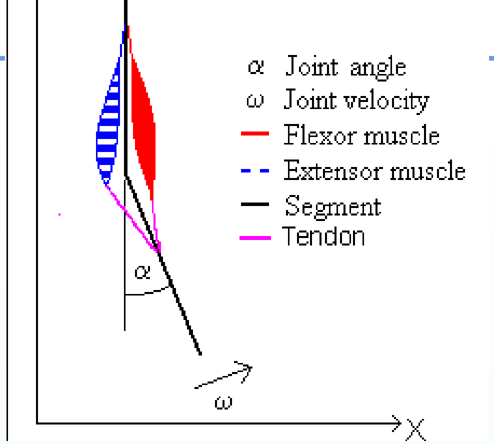
Problem description

Given a simple limb consisting of two segments (bones) two muscles (flexor and extensor) that move the lower segment around the joint axis.



Construct the model of the simple limb system for control purposes if we can measure the position of lower segment and the external forces causing contraction on the flexor and extensor muscles, respectively.

Example: a simple limb model - 2



Mechanisms

- muscle force
- muscle contraction by activation
- rotation

Modeling assumptions

F1 one mass point with mass m for the lower segment

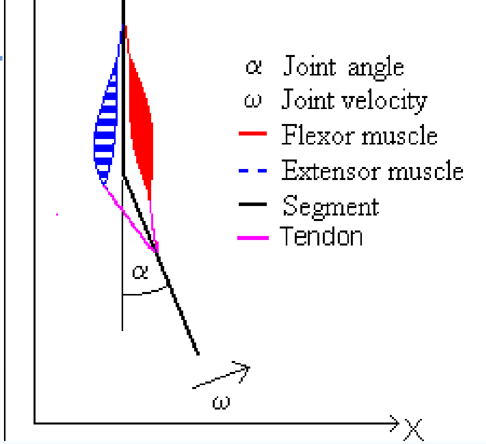
F2 the upper segment is fixed vertically

F3 fixed distance l_{COM} from the rotating axis

F4 activation state equations with fixed parameters (τ activation time, β descreasing factor)

F5 activation determines the torque of the muscles M

Example: a simple limb model - 3



Conservation balance equations:

for angular momentum $N = \Theta^{(total)} \cdot \omega$

$$\frac{dN}{dt} = M(q_1, q_2, \alpha, \omega) + m \cdot l_{COM} \cdot \cos(\alpha - \frac{\pi}{2}) \cdot g \tag{5}$$

for the acticvation states q_1 and q_2 (bioelectrical model)

$$\frac{dq_1}{dt} = \left(\frac{1}{\tau_{act}}(\beta + [1-\beta]u^f(t))\right) q_1 + \frac{1}{\tau_{act}} u^f(t) \tag{6}$$

$$\frac{dq_2}{dt} = \left(\frac{1}{\tau_{act}}(\beta + [1-\beta]u^e(t))\right) q_2 + \frac{1}{\tau_{act}} u^e(t) \tag{7}$$

Constitutive equations

- $\frac{d\alpha}{dt} = \omega$ (velocity equation)
- the actual form of $M(q_1, q_2, \alpha, \omega)$ (from biology)

Thermo-mechanical systems

Modeling of mechanical systems - 3

III. Energy-based description – simplest case: mass-point systems

Rectilinear motion: potential (position dependent) and kinetic (momentum dependent) energy - for a single mass point with mass m

$$E_{total} = V(x) + \frac{1}{2m}p^2$$

Power ($\frac{dE_{total}}{dt}$) provided by a force F: $\frac{dE_{total}}{dt} = F \cdot \frac{dx}{dt} = F \cdot v$

Rotating motion: kinetic energy for a rotating body with moment of inertia Θ

$$E_{rot} = \frac{1}{2}\Theta\omega^2$$

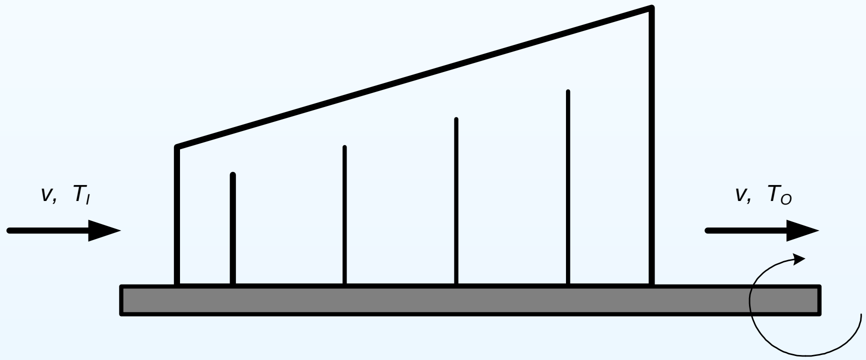
Power ($\frac{dE_{rot}}{dt}$) provided by a torque M: $\frac{dE_{rot}}{dt} = M \cdot \omega$

Equivalence with the momentum-based description

Example: simple steam turbine - 1

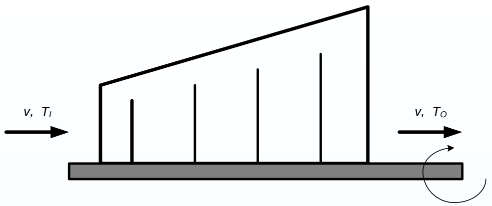
Problem description

Given a simple steam turbine that moves a rotating shaft aroung a given axis. The steam enters and leaves the turbine with a given mass flow rate v, the inlet temperature is T_I, the outlet temperature is T_O.



Construct the model of the simple turbine for control purposes if we can measure the external load torque M_{load} and the angular velocity ω.

Example: simple steam turbine - 2



Mechanisms

- thermal energy conversion
- rotation

Modeling assumptions

F1 one rotating body (shaft) with moment of inertia Θ

F2 equal and given mass flow rate v for inlet and outlet

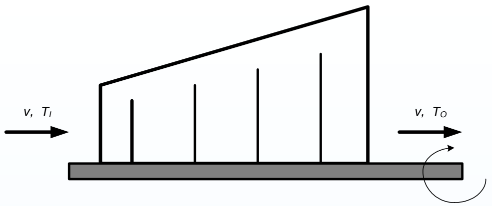
F3 given inlet T_I and outlet T_O temperatures (no condensation!)

F4 the mechanical efficiency factor χ is given and constant

F5 constant steam specific heat c_P

F6 given the external load torque M_{load} (input)

Example: simple steam turbine - 3



Conservation balance equation:
for the rotation energy

$$\frac{dE_{rot}}{dt} = P_{steam} - M_{load}\omega \qquad (8)$$

Constitutive equations

- $E_{rot} = \frac{1}{2}\Theta\omega^2$ (rotation energy)
- $P_{steam} = \chi \cdot c_P \cdot v \cdot (T_I - T_O)$ (power generated by the steam temperature drop)

Model equation with measurable quantities

$$\frac{d\omega}{dt} = \frac{1}{\Theta\omega}\left(\chi \cdot c_P \cdot v \cdot (T_I - T_O) - M_{load}\omega\right)$$