Intelligent Control Systems: Qualitative modelling

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Contents

Discrete range spaces

Universe: the range space of variables = a set of intervals

- *General qualitative*: real intervals with fixed or free endpoints

$$U_{\mathcal{I}} \;=\; \{[a_\ell, a_u] \mid a_\ell,\; a_u \in \mathcal{R},\; a_\ell \le a_u\}$$

with the **landmark set**

$$L_{\mathcal{I}} = \{a_i \mid a_i \le a_{i+1}\;,\; i \in I \subseteq \mathcal{N}\}$$

- *sign-valued case*

$$\begin{aligned} U_{\mathcal{S}} &= \{\, + \,,\, - \,,\, 0 \,;\, ? \,\} \;,\; ? \;=\; + \,\cup\, 0 \,\cup\, - \\ L_{\mathcal{S}} &= \{\, a_1 = -\infty \,,\; a_2 = 0 \,,\; a_3 = \infty\} \end{aligned}$$

- *logical* (extended)

$$U_{\mathcal{L}} \;=\; \{\; \textbf{true} \;,\; \textbf{false} \;;\; \textbf{unknown} \;\}$$

Sign algebra

Algebra over the **sign universe**

Operations: with **usual algebraic properties** (commutativity, associativity, distributivity)

- sign addition ($\oplus_S$) and substraction ($\ominus_S$)
- sign multiplication ($\otimes_S$) and division
- composite operations and functions

Specification (definition) of a sign operation is done by using **operation tables**.

Sign addition

Operation table

$a \oplus_S b$	$+$	0	$-$	$?$
$+$	$+$	$+$	**?**	$?$
0	$+$	0	$-$	$?$
$-$	**?**	$-$	$-$	$?$
$?$	$?$	$?$	$?$	$?$

Properties:

- **Growing uncertainty**
- commutative

Sign multiplication

Operation table

$a \otimes_S b$	$+$	0	$-$	$?$
$+$	$+$	0	$-$	$?$
0	0	0	0	$\mathbf{0}$
$-$	$-$	0	$+$	$?$
$?$	$?$	$\mathbf{0}$	$?$	$?$

Properties:

- **correcting** by zero values
- commutative

Interval operations – 1

Operation on intervals with *fixed* endpoints

- **Set-type definition**: the sum (or product) of two intervals $\mathcal{I}_1 = [a_{1\ell}, a_{1u}]$ and $\mathcal{I}_2 = [a_{2\ell}, a_{2u}]$ from $U_{\mathcal{I}}$ is the smallest interval from $U_{\mathcal{I}}$ which covers the interval

$$\mathcal{I}^* = \{\ b = a_1 \ \mathbf{op}\ a_2 \mid a_1 \in \mathcal{I}_1\ ,\ a_2 \in \mathcal{I}_2\}$$

- **Endpoint-type definition**: for *monotonic operations* we can compute the above as

$$\begin{aligned} E_{op} = \{\ & e_{\ell\ell} = a_{1\ell}\ \mathbf{op}\ a_{2\ell}\ ,\ e_{\ell u} = a_{1\ell}\ \mathbf{op}\ a_{2u}\ , \\ & e_{u\ell} = a_{1u}\ \mathbf{op}\ a_{2\ell}\ ,\ e_{uu} = a_{1u}\ \mathbf{op}\ a_{2u}\ \} \\ & \text{with} \quad \mathcal{I}^* = [\ min\ E_{op},\ max\ E_{op}\ ] \end{aligned}$$

  where E_{op} is formed from the endpoints

Interval operations – 2

Unusual properties caused by the fact that $\mathcal{I}^*$ *should be* ***covered*** by an interval from $U_{\mathcal{I}}$

- **growing uncertainty** with every operation
- **lack of distributivity**: the result may depend on the algebraic form *minimum number of addition is the best*

Order of magnitude intervals

Universe

- landmark set:

$$L_{\mathcal{OM}} = \{\ a_1 = -\infty\ ,\ a_2 = -A\ ,\ a_3 = 0\ ,\ a_3 = A\ ,\ a_4 = \infty\}$$

- atomic intervals:

$$LN = [-\infty, -A),\ \ SN = [-A, 0),\ \ 0 = [0, 0],\ \ SP = (0, A],\ \ LP = (A, \infty]$$

$$U_{\mathcal{OM}}\ =\ \{\ LN\ ,\ SN\ ,\ 0\ ,\ SP\ ,\ LP\ \}$$

Non-atomic intervals and operations

- pseudo-intervals: $[SP, LP] = (0, \infty]$ *or* $[LN, LP] = [-\infty, \infty]$
- operations: $LP\ \oplus_{OM}\ LN\ =\ [LN, LP]$

Order of magnitude addition

Operation table of the **order of magnitude** interval addition

$a \oplus_{OM} b$	LN	SN	0	SP	LP
LN	LN	LN	LN	$[LN, SN]$	$[LN, LP]$
SN	LN	$[LN, SN]$	SN	$[SN, SP]$	$[SP, LP]$
0	LN	SN	0	SP	LP
SP	$[LN, SN]$	$[SN, SP]$	SP	$[SP, LP]$	LP
LP	$[LN, LP]$	$[SP, LP]$	LP	LP	LP

Normalized intervals

Qualitative range space: for variables with "normal" N value

$$\mathcal{Q} = \{H, N, L, 0\}, \quad \mathcal{B} = \{0, 1\}, \quad \mathcal{Q}_E = \{H, N, L, 0, e+, e-\}$$

Intervals with non-fixed endpoints

Operation table for interval addition

$[a] + [b]$	0	L	N	H
0	0	L	N	H
L	L	N	H	$e+$
N	N	H	$e+$	$e+$
H	H	$e+$	$e+$	$e+$

This is only a possible definition!

The notion of qualitative models

The range space of the variables and parameters is interval-valued

- *sign-valued*
  - Signed Directed Graph (SDG) models
  - Confluences (sign qualitative differential equations)
- *interval-valued*
  - Qualitative Differential Equations (QDEs): constraint type, algebraic type

From AI viewpoint: qualitative models are **special knowledge representation forms** with special reasoning.

The origin of qualitative models

Engineering dynamical models in **state-space form**:

$$\begin{aligned} \frac{dx}{dt} &= f(x,u) \quad \text{(state eq.)} \\ y &= h(x,u) \quad \text{(output eq.)} \end{aligned}$$

Qualitative models can be derived *systematically* from engineering models by using

- interval-values variables and parameters
- simplified equations

The structure of a state-space model

Linearized state-space models near a *steady-state point*

$$\begin{array}{lcll} \frac{dx}{dt} & = & Ax + Bu & \text{(state eq.)} \\ y & = & Cx + Du & \text{(output eq.)} \end{array}$$

Signed structure matrices: $[A]$

$$[A]_{ij} = \begin{cases} + & \text{if} \quad a_{ij} > 0 \\ 0 & \text{if} \quad a_{ij} = 0 \\ - & \text{if} \quad a_{ij} < 0 \end{cases}$$

Structure graph

A signed directed graph $S = (V, \mathcal{E}; w)$

- **vertex set** for the state, input and output variables

$$V = X \cup U \cup Y$$
$$X \cap U = X \cap Y = U \cap Y = \emptyset$$

- **edges** for the *direct* effects between variables
- edge **weights** for the *sign* of the effect

Paths in the structure graph

A **directed path** $P = (v_1, v_2, ..., v_n)$, $v_i \in V$, $e_{i,i+1} = (v_i, v_{i+1}) \in \mathcal{E}$

- describe an indirect effect from variable v_1 to v_n
- **value** of the path

$$W(P) = \prod_{i=1}^{n-1} w(e_{i,i+1})$$

- significance of *shortest path(s)* and *directed circles*

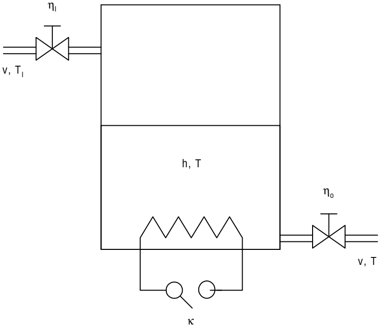
Diagnostic reasoning using SDGs

Sign-valued variables: sign of the *deviation from their steady-state value*

The effect of a variable v_i to another variable v_j

- **initial deviation** is determined by the **sign-value of the *shortest path(s)***
  - **sign-sum** is needed if not unique $\Longrightarrow$ **ambiguity**
- **steady-state effect** is the ***sign-sum* of the sign-value of *all* paths**
  - $\Longrightarrow$ **ambiguity** (often)
  - directed circles: solution of sign-linear equations

Example – Coffee machine

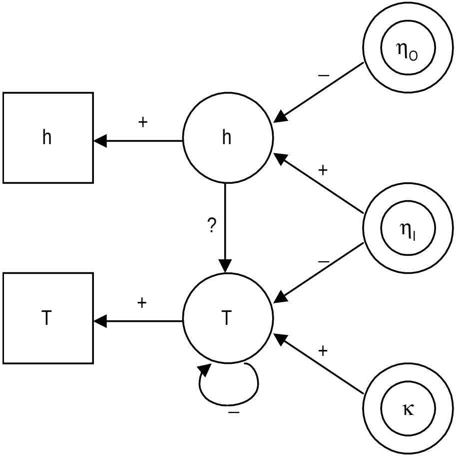


State-space model of the coffee machine

$$\begin{aligned} \frac{dh}{dt} &= \frac{v}{A}\eta_I - \frac{v}{A}\eta_O && \text{(mass)} \\ \frac{dT}{dt} &= \frac{v}{Ah}(T_I - T)\eta_I + \frac{H}{c_p \rho h}\kappa && \text{(energy)} \end{aligned}$$

t	time $[s]$
h	level in the tank $[m]$
v	volumetric flowrate $[m^3/s]$
c_p	specific heat $[Joule/kgK]$
ρ	density $[kg/m^3]$
T	temperature in the tank $[K]$
T_I	inlet temperature $[K]$
H	heat provided by the heater $[Joule/sec]$
A	cross section of the tank $[m^2]$
η_I	binary input valve $[1/0]$
η_O	binary output valve $[1/0]$
κ	binary switch $[1/0]$

SDG of the coffee machine



Origin of confluences

"Qualitative Physics" by de Kleer and Brown

Sign version of lumped nonlinear state equations (dynamic models with perfectly stirred balance volumes)

- can be *formally derived* therefrom
- *sign-valued variables and operations* are used

A complete and contradiction-free rule-set can be derived from confluences

Derivation of confluences

1. **define qualitative variables $[q]$ and δq to each of the model variables $q(t)$ as follows**:

$$q \sim [q] = sign(q) \quad , \quad dq/dt \sim \delta q = sign(dq/dt)$$

2. **operations are replaced by sign operations**, i.e.

$$+ \sim \oplus_S \quad , \quad * \sim \otimes_S \quad etc.$$

3. **parameters are replaced by $+$ or $-$ or** 0 forming *sign constants* in the confluence equations, i.e. they virtually disappear from the equations.

Solution of a confluence

In the form of an **extended truth table**
(sign-operation table)

- collect all of the *right-hand side variables* (time-dependent values!)
- enumerate all of their sign-values
- systematically enumerate all of the **possible combinations**
  $\Longrightarrow$ exponentially growing size with the number of variables

Rule generation from confluences

The **rows of the truth table of a confluence can be interpreted as a rule** if one reads them from right to left.
For example

$$\delta h = [\eta_I] \ominus_S [\eta_O]$$

with the combination $\eta_I = 0$, $\eta_I = +$ gives $\delta h = -$

$\Longrightarrow$

$$\textbf{if} \;\; (\eta_I = \textbf{closed}) \textbf{ and } (\eta_O = \textbf{open}) \;\; \textbf{then} \;\; (h = \textbf{decreasing})$$

Rule sets can be generated from the truth table of a confluence and they are complete and contradiction-free

A simple example – 1

Model equation: mass balance of the coffee machine

$$\frac{dh}{dt} = \frac{v}{A}\eta_I - \frac{v}{A}\eta_O$$

1. **qualitative variables**: $[\eta_I] \in \{0, +\}$, $[\eta_O] \in \{0, +\}$
2. all **sign constants** are "$+$"
3. **confluence**

$$\delta h = [\eta_I] \ominus_S [\eta_O]$$

A simple example – 2

Truth table of the confluence

$$\delta h = [\eta_I] \ominus_S [\eta_O]$$

δh	$[\eta_I]$	$[\eta_O]$
0	0	0
$-$	0	$+$
$+$	$+$	0
?	$+$	$+$

The derivation of discrete time qualitative DAEs

Dynamic models derived from first engineering principles: continuous time differential-algebraic equation models

- differential equations originate from conservation balances: ***to be transformed to difference equations*** (time discretization)
- selection of the ***qualitative range spaces*** of variables and parameters
- deriving the qualitative form

Qualitative signals

Qualitative range spaces

$$\mathcal{Q} = \{H, N, L, 0\}, \quad \mathcal{B} = \{0, 1\}, \quad \mathcal{Q}_{\mathcal{E}} = \{H, N, L, 0, e+, e-\}$$

with *H*igh, *L*ow, *N*ormal, *e*rror.

A qualitative signal is a signal (input, output, state and ***disturbance (fault indicator)***) that takes its values from a finite qualitative range set

An event is generated when a qualitative signal changes its value. An event e_X is formally described by a pair $e_X(t, q_X) = (t, [x](t) = q_X)$ where t is the occurrence time when the qualitative signal $[x]$ takes the value q_X.

Solution of a qualitative DAE

In the form of a **solution table**
(interval operation table)

- collect all of the *right-hand side variables* (time-dependent values!)
- enumerate all of their signal traces
- systematically enumerate all of the **possible combinations**
  $\Longrightarrow$ exponentially growing size with the number of variables

A static example: sensor with additive type fault

Algebraic model equation: $v^m = v + \chi \cdot E$
$[v] \in \mathcal{Q}$, $[v]^m \in \mathcal{Q}_e$, $\chi \in B_{-1} = \{-1, 0, 1\}$

$[v^m]$	$[\chi]$	$[v]$	mode
N	0	N	normal
H	0	H	normal
L	0	L	normal
0	0	0	normal
$e+$	1	H	faulty
H	1	N	faulty
N	1	L	faulty
L	1	0	faulty
N	-1	H	faulty
L	-1	N	faulty
0	-1	L	faulty
$e-$	-1	0	faulty

A dynamic example: mass balance of the coffee machine

Differential equation in discrete form: $h^T = h + \chi_I \cdot v - \chi_O \cdot v$
$[h], [h]^T \in \mathcal{Q}_e$, $\chi_I, \chi_O \in \mathcal{B}$ and $[v] = L$
Solution for constant inputs

$[h]^T$	$[h](t_0)$	χ_I	χ_O
(N, N, N)	N	(1,1,1)	(1,1,1)
(L, L, L)	L	(1,1,1)	(1,1,1)
...	...	...	...
(N, N, N)	N	(0,0,0)	(0,0,0)
...	...	...	...
$(e+, e+, H)$	N	(1,1,1)	(0,0,0)
$(e+, H, N)$	L	(1,1,1)	(0,0,0)
...	...	...	...
$(e-, 0, L)$	N	(0,0,0)	(1,1,1)
$(e-, e-, 0)$	L	(0,0,0)	(1,1,1)
...	...	...	...