Discrete and Continuous Dynamical Systems tutorial – Stability

1 Stability of CT-LTI systems

1.1 Stability analysis from state-space model

Simple example

Given the following CT LTI SISO state space model:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 6 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 2 & -4 \end{bmatrix} x \end{aligned}$$

1. Is the system asymptotically stable?
2. Is the system BIBO stable?

Parametric example

Given the following CT LTI SISO system:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 3 & 0 \\ 5 & p \end{bmatrix} x + \begin{bmatrix} q \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} r & 1 \end{bmatrix} x \end{aligned}$$

1. Give the parameter values p, q, r so that the above system is asymptotically stable! (If it is not possible, why?)

1.2 Stability analysis from input-output model

We are given the following differential equation:

$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 2\dot{u}(t) + u(t)$$

1. Compute the transfer function of the system.
2. Is the system asymptotically stable?
3. Is the system BIBO stable?

2 Stability of DT-LTI systems

Simple example

Given the following DT-LTI SISO system:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 4 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 1 & -3 \end{bmatrix} x(k) \end{aligned}$$

1. Is the system asymptotically stable?
2. Is the system BIBO stable?

Parametric example

Given the following DT-LTI SISO system:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.1 & p \\ q & 0.3 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 5 & 2 \end{bmatrix} x(k) \end{aligned}$$

1. Is it possible to have such a pair p, q that the system is asymptotically stable? If yes, give such a pair.
2. Give the values of the parameters p and q such that the system is jointly reachable and observable in addition of being asymptotically stable.

3 Homework:

(a) Analyse the BIBO and asymptotic stability of your continuous time system model in numerical form.

(b) Analyse the BIBO and asymptotic stability of your sampled (discrete time) system model in numerical form (using $h = 0.1$ sec sampling time).

(*) **(Supplementary)**
Compute the sampled version of your system using $h = 0.01$ and $h = 0.001$ sec sampling time. Use the Taylor series expansion of the matrix-exponential function up to the quadratic term:

$$e^{Ah} \approx I + Ah + \frac{1}{2}A^2h^2$$

How does the asymptotic and BIBO stability change with the decrease of the sampling time?

Deadline of submission: 2019.05.11. 12am
(Submit your homework as an email attachment (`hangos.katalin@virt.uni-pannon.hu`, subject: DCDS) in a hand written scanned pdf format! Please, write your name and neptun ID on the paper!)