Model building using engineering principles

Model verification
Tutorial

March 2021

1. Model structure of the coffee machine

The state space model of the coffee machine

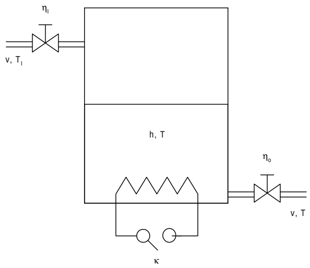
The coffee machine is a tank with a built-in on-off electrical heater. The water is fed to the tank from the tap controlled by a binary switch. The hot water flows out from the tank controlled by another binary switch.

1. ábra. The coffee machine

The flow sheet of the coffee machine is depicted in Fig. 2.

Variables and parameters in the model

t	time $[s]$	T_I	inlet temperature $[K]$
h	water level $[m]$	H	heating power $[Joule/sec]$
v_I	inlet volumetric flow rate $[m^3/s]$	v_O	outlet volumetric flow rate $[m^3/s]$
η_I	inlet binary switch $[1/0]$	η_O	outlet binary switch $[1/0]$
c_p	specific heat $[Joule/kgK]$	A	cross section $[m^2]$
ρ	density $[kg/m^3]$	T	temperature $[K]$
κ	binary heater switch $[1/0]$		



2. ábra. The flow sheet of the coffee machine

Model equations in state space model from

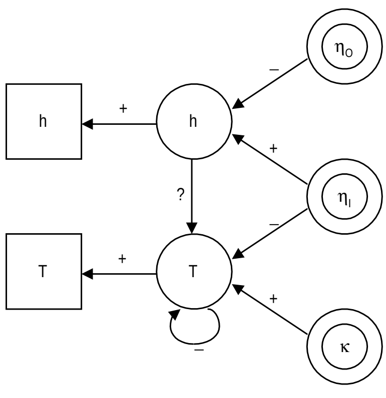
$$\frac{dh}{dt} = \frac{v_I}{A}\eta_I - \frac{1}{A}v_O \quad (1)$$

$$\frac{dT}{dt} = \frac{v_I}{A}(T_I - T)\frac{1}{h}\eta_I + \frac{H}{c_P \rho A}\kappa\frac{1}{h} \quad (2)$$

Initial conditions: $h(0) = h_0, T(0) = T_0$

Variables (signals) and parameters

- state variables (x): h, T
- input variables (u): η_I, η_O, T_I, κ
- output variables (y): h, T
- parameters: A, H, c_P, ρ, v_I, v_O, T_I



3. ábra. The structure graph of the coffee machine

2. Structural properties

Signed structure matrices

$$[A] = \begin{pmatrix} 0 & 0 \\ ? & - \end{pmatrix} \quad , \quad [B] = \begin{pmatrix} + & - & 0 \\ - & 0 & + \end{pmatrix} \quad , \quad [C] = \begin{pmatrix} + & 0 \\ 0 & + \end{pmatrix}$$

Structural properties
Consider the case, when η_I and η_O are set to constant 1, so only κ is the input variable.

- **Structural controllability**: $s - rank\ [A] = 2$ (full rank), but h is not reachable from κ via a directed path, so the model is not structurally controllable.
- **Structural observability**: $s - rank\ [A] = 2$ (full rank), and both h and T are reachable from T via a reversed directed path, so the model is structurally observable.
- The sign of the initial deviation of a step response of the output T is "+" through the shortest path $\kappa -> T -> T$

3. HOMEWORK

Task (a)
Answer the following questions related to section 2 **Structural properties** above.

- How would the situation change, if η_O were the only input variable?
- How would the situation change, if η_I were the only input variable?
- What is the sign of the initial deviation of a step response of the output h when κ is the only input variable?

Task (b)
Consider an open tank that has a free (gravitation) outflow with the volumetric flow rate $v_{ki} = Kh$. The tank contains water with constant temperature. The water is fed into the tank using a pump with controllable speed through a binary valve. The water outflow is controlled by a binary valve, too. Assume a hole on the wall of the tank at a height h^*, through which the water flows out with free (gravitation) outflow.

You have already developed a simple model of this system in an earlier homework.

Determine the signed structure matrices of the linearized model and construct the structure graph.
What are the structural properties of this model?