Dynamic system modeling for control and diagnosis

Modelling fundamentals

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Contents

The general modelling problem

- **Given**: the system to be modelled, modelling goal
- **Contstruct**: a mathematical model that describes the behaviour of the system

$$\frac{dh}{dt} = \frac{v_{be}}{A} sz_{be} - \frac{v_{ki}}{A} sz_{ki}$$

$$\frac{dT}{dt} = \frac{v_{be}}{Ah}(T_{be} - T) sz_{be} + \frac{Q_H}{c_p \rho A h} k$$

$$h(0) = h_0 \ , \ T(0) = T_0$$

The modelling goal

Problem description: a pair of the system and the **modelling goal**
Modelling goal

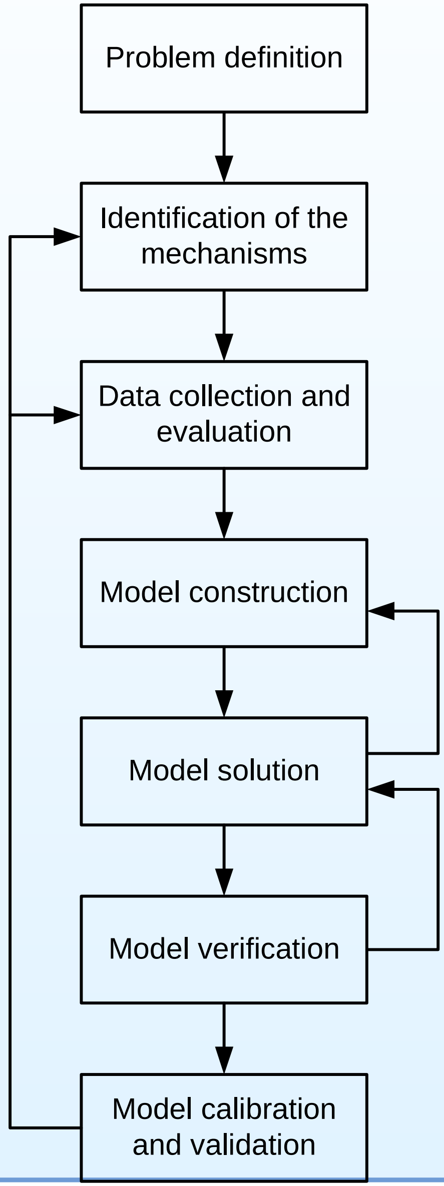
- possibilities and categories
  - dynamic and static (steady-state) simulation
  - system design
  - **process control** (prediction, regulation, identification, diagnosis)
- determines the (vailidity) domain of the model
- influences the following model properties
  - which mechanisms should be taken into account
  - the mathematical form (algebraic equations, differential equations, graphs etc)
  - the accuracy (of the characteristic variables)

The 7 steps modelling procedure - 1

1 Problem definition
formal description
- system definition
- modelling goal determination
- flowsheet construction (equipments, variables)

2 Mechanisms identification
- collection of phenomena
(e.g. convection, transfer, reaction, evaporation)

3 Data collection and evaluation
- constants from data tables (accuracy!)
- properties of equipments and operation
- measured data (preliminary)



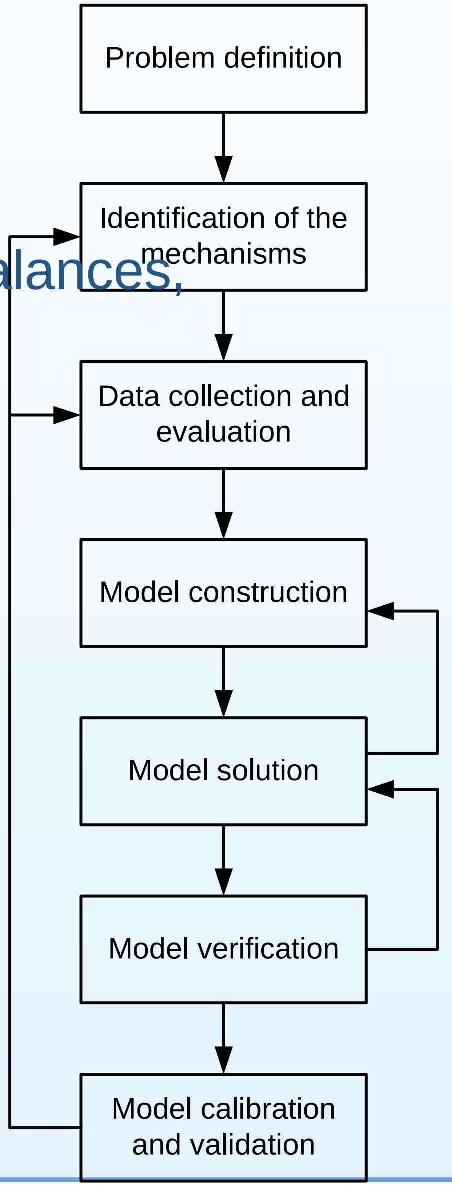
The 7 steps modelling procedure - 2

4 Model construction

- determination of balance volumes
- formulation of modelling assumptions
- construction of model equations (conservation balances, constitutive equations)
- determination of initial and boundary conditions

5 Model solution

- implementation or
  recasting of solution method
- model checking (plausibility and accuracy)



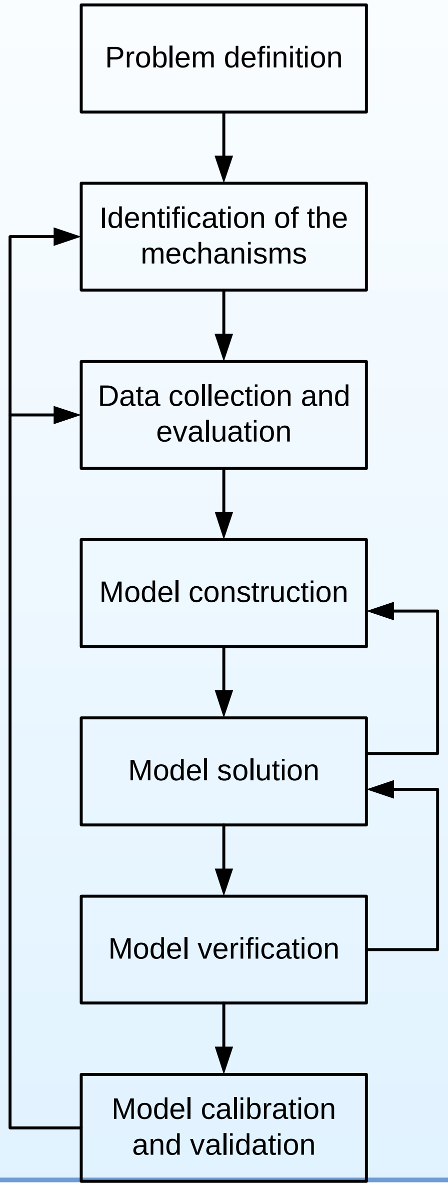
The 7 steps modelling procedure - 3

6 Model verification

- verifying qualitative model
  behaviour against engineering intuition
- checking dynamic properties (e.g. stability)
  on the model

7 Model calibration and validation

- model calibration
  estimating unknown/uncertain model parameters
  using measured data
- model validation
  comparing the model and the real system
  (measured data) using statistical methods



Mechanisms - phenomena

These depend on the type of the physical systems:

- mechanical systems
- thermodynamical (process, energy) systems
- electrical systems
- chemical, biological, etc. systems

Most important mechanisms in thermodynamical systems

- flows: convective, diffusive
- heating, cooling
- mass and energy transfer
- phase transitions (evaporation, boiling melting, ...)

Conservation balances - 1

Balance volumes: for constructing conservation balances

- most often with *constant volume*
- *perfectly stirred* (concentrated parameter, the balance is in the form of ordinary differential equations)

Conserved (extensive) **quantities**:

- ovarall mass
- energy (entalpy, internal energy)
- component mass, (momentum)

Dynamic conservation balance in general form: for a conserved quantity

$$\left\{ \begin{array}{c} rate\ of \\ change \end{array} \right\} = \left\{ \begin{array}{c} in\text{-} \\ flow \end{array} \right\} - \left\{ \begin{array}{c} out\text{-} \\ flow \end{array} \right\} + \left\{ \begin{array}{c} source \\ sink \end{array} \right\}$$

Intensive quantities

Intensive quantities **equilibrate** when joining sub-systems

- potential (driving force) type quantities
- drive flows and transfer (usually linear approximation without any cross-effects)
- measurable quantities
- extensive - intensive pairs
  - overall mass m - pressures p
  - energy U - temperature T
  - component mass m_X - concentration c_X (chemical potential)

Extensive - intensive relationships

- $U = c_P m T$ (c_P specific heat)
- $m_X = \frac{m}{\rho} c_X$ (ρ density)

Conservation balances - 2

Dynamic conservation balance for **overall mass**

- ***no source/sink***
- the overall mass m is measurable (e.g. level measurement)
- for perfectly stirred balance volumes the in- (v_B) and out-flows (v_K) are mass flows $[kg/s]$

Example:

$$\frac{dm}{dt} = v_B - v_K$$

Conservation balances - 3

Dynamic conservation balance for **energy**

- source/sink: external (e.g. electrical) heating/cooling or heat transfer Q ($[J/s]$)
- for perfectly stirred balance volumes the in- ($c_{pB} v_B T_B$) and out-flows ($c_P v_K T$) are energy flows $[J/s]$
- The energy U is directly ***not*** *measurable*, we use the temperature instead in the equation => transformation into intensive form

Example:

$$\frac{dU}{dt} = c_{pB} v_B T_B - c_P v_K T - Q$$

Constitutive equations

Further equations necessary to complete the model

- usually algebraic equations
- most common types:
    - extensive-intensive relationships
    - transfer rate equations
    - termodinamical reltionships
    - balance volume relations
    - equipment and control relations

Modelling assuptions

The list should be ***collected incrementally*** during the modelling procedure

Most common modelling assumption types:

- assumptions on the time-dependent behaviour of the (sub)system/mechanisms
  (e.g. dynamic, steady-state)
- assumptions on the balance volumes
  (e.g. only fluid phase, vapour and liquid phase)
- assumption on the spatial distributions
  (e.g. perfectly stirred/concentrated parameter)
- assumptions on the presence/absense or properties of mechanisms
  (e.g. no evaporation, linear heat transfer)
- assumptions on the negligible effects
  (e.g. density depends only on T, specific heat c_P is constant)
- assumptions on the required domain of state variables, and on the required accuracy

Ingredients of a modell

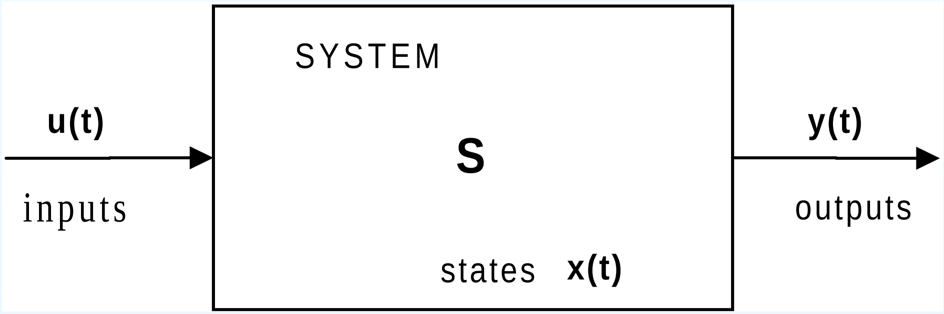
- System description (flowsheet, variables)
- Modelling goal
- Mechanisms
- Modelling assumptions
- Model data (data, unit, source, accuracy)
- Balance volumes (indicated on the flowsheet)
- Model equations (conservation balance equations, constitutive equations, initial and boundary conditions)
- Model variables and parameters

State-space model form

System (**S**): operates on signals (time-dependent, variables)

$$y = \mathbf{S}[u]$$

- inputs (u) and outputs (y); states (x)



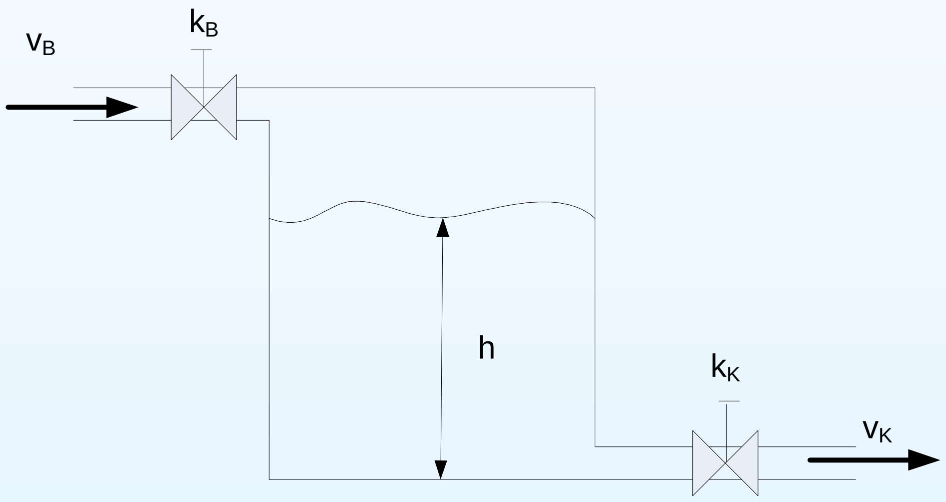
Signals in a state-space model originates from first engineering principles

- state variables (x): conserved extensive quantities (or their intensive pairs)
- input variables (u): appear on the right-hand sides of the differential equations, manipulable (measurable)
- output variables (y): measurable, not directly manipulable (state variable or depends therefrom)

Example: tank with gravitational outflow

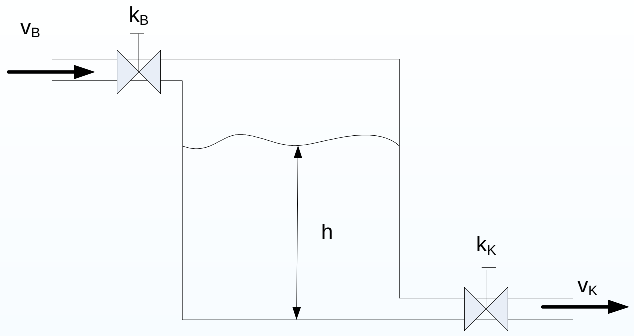
Problem description

Given a tank with constant cross section that is used for storing water. The water flows into the tank through a binary input valve, the outflow rate is driven by gravitation, i.e. depends on the water level in the tank, but it is controlled by a binary output valve.



Construct the model of the tank for diagnostic purposes if we can measure the water level and the status of the valves.

Example: tank with gravitational outflow - 2



Mechanisms

- in- and out-flow
- gravitational outflow (driven by the hydrostatic pressure)

Modelling assumptions

F1 one balance volume (the tank) perfectly stirred

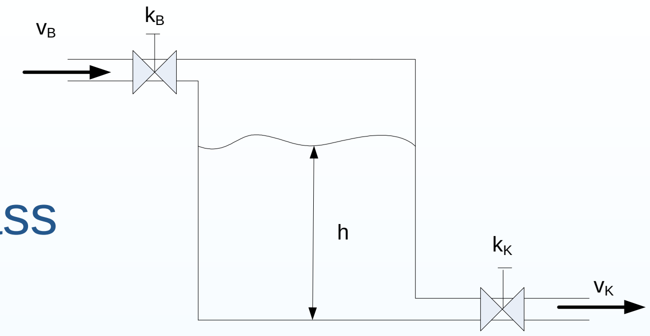
F2 only water is present (only overall mass balance is considered)

F3 gravitational outflow

F4 constant cross-section A

F5 density (ρ) is constant

Example: tank with gravitational outflow - 3



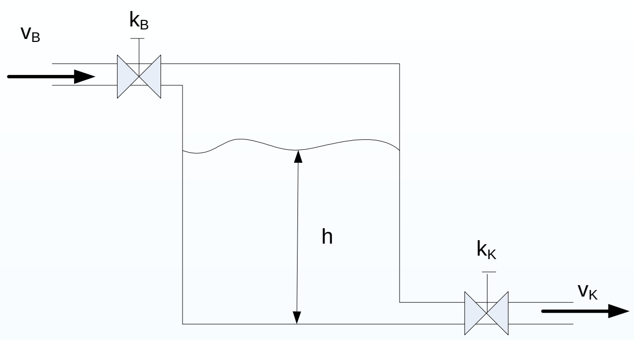
Conservation balance equation: for overall mass

$$\frac{dm}{dt} = v_b - v_k \quad (1)$$

Constitutive equations

- $m = A \cdot h \cdot \rho$ (water level h is measurable)
- $v_B = v_B^* k_B$ (valve status k_B is measurable)
- $v_K = K \cdot h \cdot k_K$ (gravitational outflow, valve status k_K is measurable)

Example: tank with gravitational outflow - 4



Model equation with measurable variables:

$$\frac{dh}{dt} = \frac{v_b^*}{A\rho} k_b - \frac{K}{A\rho} h \cdot k_K \qquad (2)$$

State-space model form

- state variable: water level h
- input variables: status of the valves k_B and k_K
- output variable: water level h