Model building using engineering principles

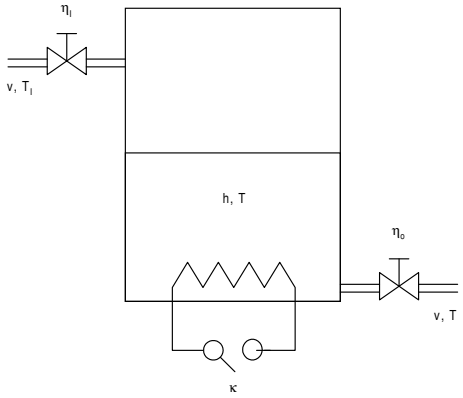
Building of simple models
Tutorial

22th March 2019

1. Modelling of a coffee machine

Modelling task

The coffee machine is a tank with a built-in on-off electrical heater. The water is fed to the tank from the tap controlled by a binary switch. The hot water flows out from the tank controlled by another binary switch. The flow sheet of the coffee machine is depicted in Fig. 1. The task is to build a dynamic model of the coffee machine for



1. ábra. The flow sheet of the coffee machine

diagnostic purposes when we can measure the water temperature and the water level.

Solution

Flow sheet is seen in Fig. 1.

Modelling assumptions

1. Only the water heating tank is described
2. Perfectly stirred

3. Constant physico-chemical proprties
4. The cross section (A) of the tank is constant
5. Constant heat power H controlled by a binary switch (κ)

Variables and parameters in the model

t	time $[s]$	T_I	inlet temperature $[K]$
h	water level $[m]$	H	heating power $[Joule/sec]$
v_I	inlet volumetric flow rate $[m^3/s]$	v_O	outlet volumetric flow rate $[m^3/s]$
c_p	specific heat $[Joule/kgK]$	A	cross section $[m^2]$
ρ	density $[kg/m^3]$	T	temperature $[K]$
κ	binary heater switch [1/0]		

Model equations **Dynamic conservations balances** for the water

- *mass balance*

$$\frac{dM}{dt} = \rho v_I - \rho v_O \tag{1}$$

- *energy balance*

$$\frac{dE}{dt} = c_P \rho T_I v_I - c_P \rho T v_O + \kappa H \tag{2}$$

Constitutive equations

$$M = \rho A h \tag{3}$$
$$E = c_P \rho A h T \tag{4}$$

We substitute the constitutive equations into the conservation balances and perform the differentiation. We also substitute the transformed form of Eq. (1) into Eq. (2). Finally we obtain the following two ODEs:

$$\frac{dh}{dt} = \frac{1}{A} v_I - \frac{1}{A} v_O \tag{5}$$
$$\frac{dT}{dt} = \frac{1}{A} v_I T_I \frac{1}{h} - \frac{1}{A} v_O T \frac{1}{h} + \frac{H}{c_P \rho A} \kappa \frac{1}{h} \tag{6}$$

Initial conditions: $h(0) = h_0, T(0) = T_0$

Parameters : A, H, c_P ρ

Variables (signals)

- state variables (x): h, T
- input variables (u): v_I, v_O, T_I, κ
- output variables (y): h, T

2. HOMEWORK

Consider an open tank that has a free (gravitation) outflow with the volumetric flow rate $v_{ki} = Kh$. The tank contains water with constant temperature. The water is fed into the tank using a pump with controllable speed through a binary valve. The water outflow is controlled by a binary valve, too.

(a) Construct a simple dynamic model of the fault-free tank for diagnostic purposes.

(b) Assume a hole on the wall of the tank at a heigh h^*, through which the water flows out with free (gravitation) outflow. Construct a simple dynamic model of the tank with a hole using the model developed in point (a).