Model building using engineering principles

Building of simple energy models
Tutorial

5th April 2019

1. A simple heat exchanger model

Heat exchangers are one of the simplest units in power industries – they can be found in almost every plant. As their name suggests, heat exchangers are used for energy exchange between at least two fluid phase (gas or liquid) streams, a hot and a cold stream.

The simplest model of a heat exchanger describes a so called *heat exchanger cell.* A heat exchanger cell is a primitive dynamic unit which consists of two perfectly stirred (lumped) balance volumes (called *lumps*) connected by a heat conducting wall. The lumps with their variables are shown in Fig. 1.

1.1. Modeling assumptions

In order to obtain a simple model with only two state equations, the following simplifying modeling assumptions are used:

F1 Constant volume and mass hold-up is assumed in both lumps ($j = c, h$).

F2 Constant physico-chemical properties are assumed for the density: ρ_j and the specific heat: c_{Pj} in both lumps, i.e. for $j = c, h$.

F3 Constant heat transfer coefficient (K_T) and area (A) is considered.

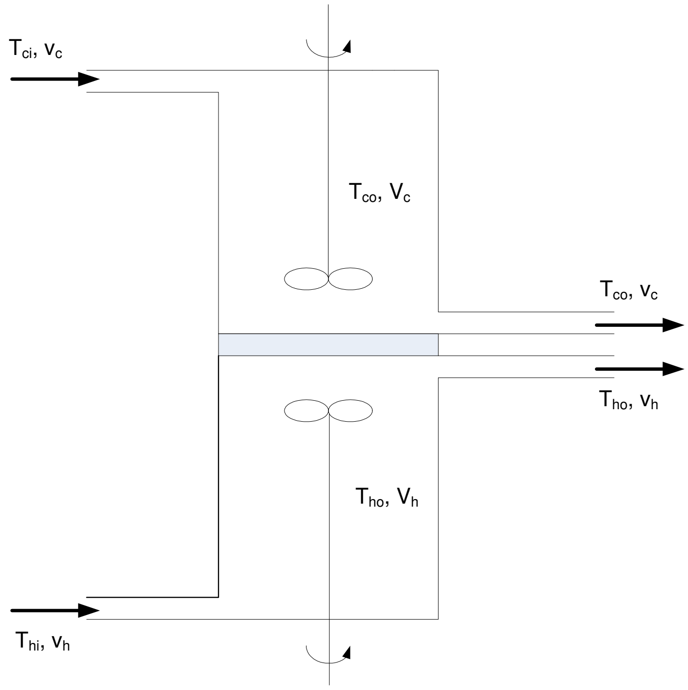
F4 We assume completely observable states, i.e. $y(t) = x(t)$.

1.2. Conservation balances

The continuous time state equations of the heat exchanger cell above are the following energy conservation balances in their intensive variable form:

$$\dot{T}_{co}(t) = \frac{v_c(t)}{V_c}(T_{ci}(t) - T_{co}(t)) + \frac{K_T A}{c_{pc}\rho_c V_c}(T_{ho}(t) - T_{co}(t)) \tag{1}$$

$$\dot{T}_{ho}(t) = \frac{v_h(t)}{V_h}(T_{hi}(t) - T_{ho}(t)) + \frac{K_T A}{c_{ph}\rho_h V_h}(T_{co}(t) - T_{ho}(t)) \tag{2}$$



1. ábra. A heat exchanger cell

where T_{ji} and T_{jo} are the inlet and cell temperatures (that are equal to the outlet temperatures because of the perfectly stirred assumption), V_j is the volume and v_j is the volumetric flow rate (measured in units $[m^3/s]$) of the two sides ($j = c, h$) respectively.

1.3. System variables

The *state vector* is therefore composed of the two outlet temperatures:

$$x_1 := T_{co}, \quad x_2 := T_{ho} \tag{3}$$

There are a number of possibly time-dependent variables on the right-hand side of the above equations which may act as manipulable *input variables* or disturbances, depending on the measurement and actuator settings and on any additional modeling assumptions we may have. These are as follows:

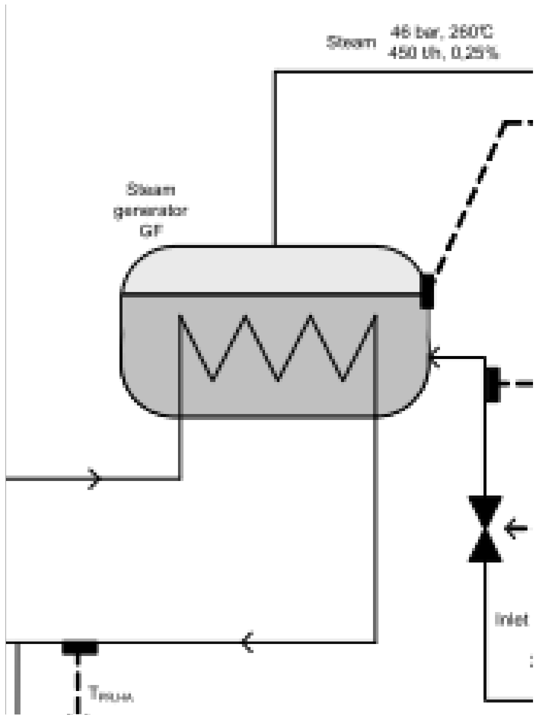
- the inlet temperatures: T_{ci} and T_{hi},
- the volumetric flowrates: v_c and v_h.

The special cases of the heat exchanger cell models are obtained by specifying assumptions on their variation in time. For every case, the output equation is

$$y(t) = x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{4}$$

2. A simple steam generator model

The steam generators connect the primary and secondary circuit and transfer the energy generated by the reactor to the secondary steam flow. The steam generator itself is a vessel that contains water of the secondary circuit. The vapor is produced from this secondary circuit water by heating the liquid phase using the energy coming from the circulating primary circuit liquid as it can be seen in Fig. 2.



2. ábra. Simplified flowsheet of the steam generator

2.1. Modeling assumptions

Because the focus of our model is the primary circuit and its controllers, the following simplifying assumptions are made for the steam generators:

1. The dynamics of the primary side of the steam generators is very quick compared to that of the secondary side, therefore it is assumed to be in a quasi steady state and no conservation balances are constructed for it.

2. The dynamics of the secondary side vapor phase in the steam generators is also assumed to be very quick compared to that of the secondary side liquid, equilibrium is assumed between the water and the vapor phases.

3. Constant physical properties are assumed for the secondary side of the steam generators.

4. All the controllers acting on the secondary side (including the liquid level controller and the secondary steam pressure controller) are assumed to be ideal.

2.2. Conservation balances

There is only a single balance volume in the steam generators, the liquid of the secondary side, where the overall mass balance is simplified to an algebraic equation $M_{SG} = const$ (M_{SG} is the liquid mass), because the inlet secondary water mass flow rate $m_{SG,SW}$ and the outlet secondary steam mass flow rate $m_{SG,SS}$ is kept to be equal by the ideal water level controller of the steam generators

$$m_{SG,SW} = m_{SG,SS} = m_{SG}$$

Then the energy balance for the secondary water in the steam generators is in the form

$$\begin{aligned} \frac{dU_{SG}}{dt} &= c^L_{p,SG} m_{SG} T_{SG,SW} - c^V_{p,SG} m_{SG} T_{SG} - m_{SG} E_{evap,SG} + \\ &\quad + K_{T,SG}(T_{PC} - T_{SG}) - W_{loss,SG} \end{aligned} \tag{5}$$

where U_{SG} is the internal energy, $c^L_{p,SG}$ is the water specific heat, $c^V_{p,SG}$ is the vapor specific heat, $T_{SG,SW}$ is the inlet temperature, T_{SG} is the temperature, $E_{evap,SG}$ is the evaporation energy, and $W_{loss,SG}$ is the heat loss.

2.3. Constitutive equations

The algebraic constitutive equations describe the relationships between physical properties and temperature:

$$U_{SG} = c^L_{p,SG} M_{SG} T_{SG} \tag{6}$$

$$p_{SG} = p^T_*(T_{SG}) \tag{7}$$

where p_{SG} is the pressure, and p^T_* is a quadratic function that approximates the values obtained from the steam table.

3. HOMEWORK

Consider a cup of hot water with a given initial mass M_0 and initial temperature $T_0 \leq 100^oC$ that is standing on a table in a large perfectly stirred room of constant ambient temperature T^0.

(a) Construct a simple dynamic model of the fault-free cup with water for diagnostic purposes.

(b) Assume a hole on the wall of the cup at a heigh h^*, through which the water flows out with free (gravitation) outflow. Construct a simple dynamic model of the tank with a hole using the model developed in point (a).