CCS tutorial
Mathematical statistics, Parameter estimation

1 Vector valued random variables

Let us given two scalar-valued Gaussian random variables $\eta_1 \sim \mathbb{N}(1, 4)$, and $\eta_2 \sim \mathbb{N}(2, 16)$.

1. Plot the probability density functions f_{η_1} and f_{η_2} of random variables η_1 and η_2 in the same coordinate system!

2. Let us assume that the random variables η_1 and η_2 are independent and form a vector valued random variable $\eta = [\eta_1, \eta_2]^T$ from them.

   - Which type of distribution does the vector valued random variable η have?
   - Compute the mean value and the variance of the vector valued random variable η.

3. (HOMEWORK)
   Let us assume that the random variables η_1 and η_2 have a covariance $COV(\eta_1, \eta_2) = 2.3$ and form a vector valued random variable $\eta = [\eta_1, \eta_2]^T$ from them.

   - Which type of distribution does the vector valued random variable η have?
   - Compute the mean value and the variance of the vector valued random variable η.

2 Parameter estimation

1. Consider a scalar valued random variable ξ and let us have a measured data set about it

$$D(5) = \{0.5,\ -0.6,\ 0.3,\ -0.2,\ 0.0\}$$

   - Compute an estimate of the mean value of ξ.
   - Compute an estimate of the variance of ξ.
   - Could the measured data be independent? Compute an estimate of $r_{\xi\xi}(1)$.

2. Consider the following model that is linear in parameters:

$$y^{(M)} = px$$

   - How many parameters does this model have?
   - Consider a measured data set consisting of (y_i, x_i) pairs

     $D_1(5) = \{(0.5, 1.0),\ (0.6, 1.0),\ (0.3, 1.0),\ (-0.2, 1.0),\ (0.5, 1.0)\}$

     Compute an estimate of p if possible with its mean value and variance.

3. Consider the following model that is linear in parameters:

$$y^{(M)} = \sum_{j+1}^{2} p_j x_j$$

   - Consider a measured data set consisting of (y_i, x_{i1}, x_{i2}) values

     $D_2(4) = \{(0.5, 1.0, 1.0),\ (0.6, 1.0, 1.0),\ (0.3, 1.0, 1.0),\ (-0.2, 1.0, 1.0)\}$

     Compute an estimate of p if possible with its mean value and covariance matrix.

- How could you improve the situation? Design a new measurement that makes the estimation possible.

- How many measured data set is needed in the minimal case? Which measurements could you leave out from the data set to still have an estimate.

4. Homework
   Consider a scalar valued random variable ξ and let us have a measured data set about it

$$D(5) = \{0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5\}$$

- Compute an estimate of the mean value of ξ.

- Compute an estimate of the variance of ξ.

- Could the measured data be independent? Compute an estimate of $r_{\xi\xi}(1)$.