CCS tutorial
Random variables, stochastic processes, Stochastic DT-LTI models

1 Random variables

Let us given the scalar-valued Gaussian random variable $\xi \sim \mathbb{N}(1,4)$, and the vector valued random variable $\eta \sim \mathbb{N}(m_\eta, \Delta_\eta)$ with

$$m_\eta = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad , \quad \Delta_\eta = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

1. Plot the probability density functions $f_\theta(x)$ and $f_\chi(x)$ of random variables θ and χ in the same coordinate system, where θ belongs to a normal distribution with expected value 0 and variance 1 and χ belongs to a normal distribution with expected value 1 and variance 5!

2. Compute the mean value and the variance of the transformed random variable $\tilde{\xi} = 2\xi + 1$, where ξ is given above. Is the transformed random variable normally distributed?

3. Consider the vector-valued random variable η above.

   - Are its elements, i.e. the scalar-valued random variables η_1 and η_2 independent?
   - Compute the mean value and the variance of the transformed random variable
   $$\tilde{\eta} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \eta + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
   - Is the transformed random variable normally distributed?

2 Stochastic processes

1. Given a scalar-valued white noise stochastic process $\{e(k)\}_{-\infty}^{\infty}$ with variance σ^2. Let us construct from it a stochastic process by the equation
$$y(k) = e(k) + 0.5e(k-1) + 0.6e(k-2) + 0.7e(k-3)$$
   - What kind of process is the stochastic process $\{y(k)\}_{-\infty}^{\infty}$?
   - Compute the mean value function $m_y(k)$ and the (auto)covariance function $r_{yy}(k)$ of the stochastic process $\{y(k)\}_{-\infty}^{\infty}$.

2. Consider the following stochastic process:
$$w(k) = z(k) + 0.1z(k-1) + 0.8z(k-3)$$
where z is a sequence of independent scalar valued random variables with the same distribution, $E(z(k)) = 0$, and $D(z(k)) = \sigma$, for every k:
   - What kind of process is the stochastic process $w(k)$ and $z(k)$?
   - Compute the (auto)covariance function $r_{ww}(k)$ for $k = 1, 3, -2$

3. Consider the following two moving-average (MA) processes:
$$\begin{aligned} z(k) &= e(k) + 0.6e(k-1) + 0.1e(k-2) \\ y(k) &= e(k) + 0.3e(k-1) + 0.8e(k-2) \end{aligned}$$
where $\{e(k)\}_{-\infty}^{\infty}$ is a discrete time white noise process with variance $D^2(e(k)) = \sigma^2$
   - Compute the cross-covariance function $r_{zy}(k)$ $\forall k$.

4. Homework
Given a scalar-valued white noise stochastic process $\{e(k)\}_{-\infty}^{\infty}$ with variance σ^2. Let us construct from it a stochastic process by the equation
$$y(k) = e(k) - 0.2e(k-1)$$

- What kind of process is the stochastic process $\{y(k)\}_{-\infty}^{\infty}$?
- Compute the mean value function $m_y(k)$ and the (auto)covariance function $r_{yy}(k)$ of the stochastic process $\{y(k)\}_{-\infty}^{\infty}$ for the values $k = 0, \pm 1, \pm 2, \pm 3, ...$!
- Compute the cross-covariance function $r_{ye}(k)$ for the values $k = 0, \pm 1, \pm 2, \pm 3, ...$!