CCS tutorial
Sampling, DT-LTI models

1 Matrix functions

Given the following real square matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad , \quad Q = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \tag{1}$$

1. Compute the following quantities:

$$Q^{-1} \quad , \quad A^{-1}$$

2. Compute (t is a constant parameter)

$$e^{Q} \quad , \quad e^{2Q} \quad , \quad e^{Qt} \quad , \quad e^{A} \quad , \quad e^{At}$$

2 Sampling

1. Given the following CT-LTI system

$$\begin{aligned} \dot{x} &= -3.2x + 2.5u \\ y &= 0.2x \end{aligned}$$

   - How many inputs, states and outputs does the system have?
   - Discretize the system if the sampling time is 2 sec, i.e. $h = 2\ s$!

2. Given the following CT-LTI system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} -1 & 1 \end{bmatrix} x \end{aligned}$$

   - How many inputs, states and outputs does the system have?
   - Discretize the system if the sampling time is 1 sec, i.e. $h = 1\ s$!

3 DT-LTI state space and input-output models

1. A DT-LTI system is given by following SISO input-output model:

$$2y(k+3) + 6y(k+2) - 2y(k) = u(k+1) + 3u(k) \tag{2}$$

   - Compute its pulse transfer operator!
   - Give a possible state space model of this DT-LTI system!
   - Is this state space model unique? Why?
   - Give the pulse response function of the system!

2. Show that the Markov parameters $\mathcal{M}_i = C\Phi^i\Gamma$ are independent from the representation of the state space model, i.e. it is invariant with respect to coordinate transformation of the state space model!

3. Given the following DT-LTI system with its state space model

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 4 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 1 & 2.5 \end{bmatrix} x(k) \end{aligned} \tag{3}$$

   with the initial condition

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

   - How many inputs, states and outputs does the system have?

- Assuming $u(k) = 0, \; k = 0, 1, 2,$ compute the state vector at $k = 1, 2$.
- Compute the pulse transfer operator of the system!
- Give the input-output model in difference equation form!
- Give an equivalent state space representation with the transformation matrix
$$T = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$
- Give the Markov parameters of the original and the transformed system!
- Give the pulse response function of the system!