CCS tutorial-Mathematical basics

1 Vectors and matrices

Given the following real vectors and matrices:

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad , \quad y = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \tag{1}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \tag{2}$$

1. Compute the following transposed quantities:

$$x^T \quad , \quad y^T \quad , \quad A^T \quad , \quad B^T$$

2. Compute when possible

$$A \cdot x \quad , \quad B \cdot x \quad , \quad A \cdot y \quad , \quad B \cdot y$$

3. Compute when possible

$$x^T \cdot x \quad , \quad x^T \cdot A \cdot x \quad , \quad x^T \cdot B \cdot x \quad , \quad y^T \cdot x$$

4. Compute the **eigenvalues** of the matrices A and B.

5. Compute the **inverse matrix** of

   - matrix A
   - matrix B

2 Complex numbers

Complex numbers: $a \in \mathbb{C}$, $a = \alpha + \mathbf{j}\beta$ with $\mathbf{j} = \sqrt{-1}$

- $|a| = r = \sqrt{\alpha^2 + \beta^2}$

- **polar (trigonometric) form**: $a = r\left(\cos(\Omega) + \mathbf{j}\sin(\Omega)\right)$

- **exponential form**: $a = re^{\mathbf{j}\Omega} = r\left(\cos(\Omega) + \mathbf{j}\sin(\Omega)\right)$

Given the following complex numbers

$$a = 2 + 3\mathbf{j} \quad , \quad b = -2\mathbf{j} \quad , \quad c = 1 \quad , \quad d = 5\left(\cos\left(\frac{\pi}{2}\right) + \mathbf{j}\sin\left(\frac{\pi}{2}\right)\right)$$

1. Place a, b, c, d on the complex plane

2. Compute the polar and exponential form of a, b, c, d.

3 Complex eigenvalues

Compute the eigenvalue of the matrix

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$$

4 Solution of a differential equation

1. Give the solution of the following ordinary differential equation

$$\frac{dx(t)}{dt} = -3x(t) \quad , \quad x(0) = 1$$

What is $\lim_{t \to \infty} x(t)$?

2. Give the solution of the following ordinary differential equation

$$\frac{dx(t)}{dt} = (-3 + 3\mathbf{j})x(t) \quad , \quad x(0) = 1$$

What is $\lim_{t \to \infty} x(t)$?