Model building using engineering principles
Modelling of electrical systems

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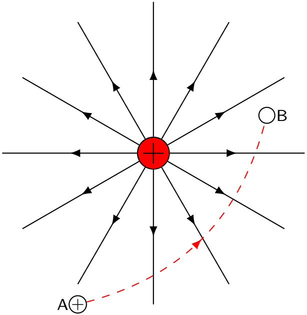
Contents

Voltage

- Potential difference between two point
- Amount of energy required to move a unit of positive charge from a lower to a higher potential
- $1V = 1J/1C$
- Always measured between two points
- Direction: from (+) to (-)



In a **static** electric field:

$$V_{AB} = V_{x_B} - V_{x_A} = -\int_{x_A}^{x_B} \vec{E} d\vec{l}$$

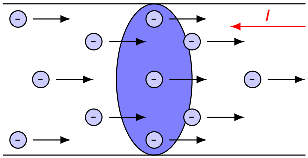
(independent of the path)

Current

- Amount of electric charge flowing through a specific point in one second

$$I = \frac{Q}{t}, \quad 1A = \frac{1C}{1s}$$

- The physical direction of current is determined by the negatively charged electrons
- By convention, positive current is the opposite direction of the flow of electrons



Characterization of electrical systems

- Characterization: through modeling objects
- Modeling objects: two-poles
    - poles: input and output ports
    - connect to the environment
    - through which electrical current may flow
- Electrical network: set of connected two-poles
    - simplest electrical system
    - concentrated parameter system model

Active and passive elements

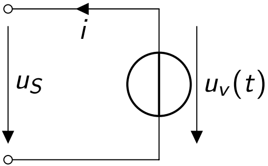
- Basic active elements
  - energy sources
  - current source, voltage source
- Basic passive elements
  - **not sources** of energy
  - resistor, capacitor, inductor
  - capacitors and inductors can *store* energy, but some external energy source is needed to charge the capacitor / establish the current of the inductor
- Sign conventions
  - Current and voltage of active elements have opposite signs
  - Current and voltage of passive elements have the same sign

Sources

Voltage source

- ideal voltage source
  - no internal resistance
  - produces constant voltage

$$u_v(t) = u_S$$



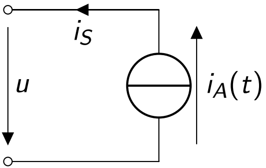
- other types
  - AC voltage source $u_V(t) = U \sin(\omega t)$
  - controlled voltage source $u_V(t) = f(i(t)),\ u_V(t) = f(u(t))$
  - nonlinear voltage source $u_V(t) = g(t)$

Sources

Current source

- ideal current source
  - produces constant current

$$i_A(t) = i_S$$



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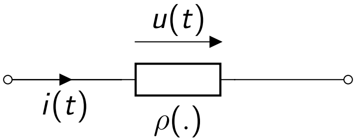
Resistors

- A two-pole is called a **resistor** if there is a static relationship between its voltage ($u(t)$) and current ($i(t)$)values.

$$u(t) = \rho(i(t))$$

- $\rho(.)$ is the *resistance function*



Resistors

- Types of resistors:
  - linear: $u(t) = R(t) \cdot i(t)$, $R(t)$ is the *resistance*
  - linear and time invariant: $R(t) = R$ constant resistance
- The inverse of the resistance function is called the *conductance function* (γ)

$$i(t) = \gamma(u(t))$$

- The reciprocal value of the resistance is called *conductance* (G)

$$i(t) = G(t) \cdot u(t)$$

$$i(t) = G \cdot u(t), \quad G = \frac{1}{R}$$

Capacitors

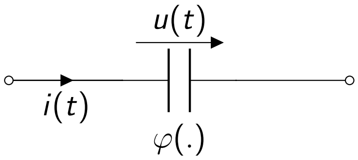
- A two-pole is called a **capacitor** if its dynamic behavior can be described by the equations

$$i(t) = \frac{dq(t)}{dt}$$

and

$$q = \varphi(u) \quad \text{or} \quad u = h(q)$$

- $q(t)$ is the electrical charge
- φ is the characteristic function of the capacitor



Capacitors

- Types of capacitors
  - general:
$$q(t) = \varphi(u(t))$$
  - linear:
$$q(t) = C(t) \cdot u(t)$$
    $C(t)$ is the *capacitance*
  - linear time-invariant (constant capacitance):
$$C(t) = C$$
- LTI case:
$$i(t) = C \cdot \frac{du(t)}{dt}$$

Inductors

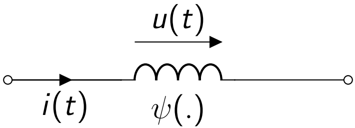
- A two-pole is called an **inductor** if its dynamic behavior can be described by the equations

$$u(t) = \frac{d\Psi(t)}{dt}$$

and

$$\Psi = \psi(i) \quad \text{or} \quad i = h(\Psi)$$

- $\Psi(t)$ is the magnetic flux
- ψ is the characteristic function of the inductor



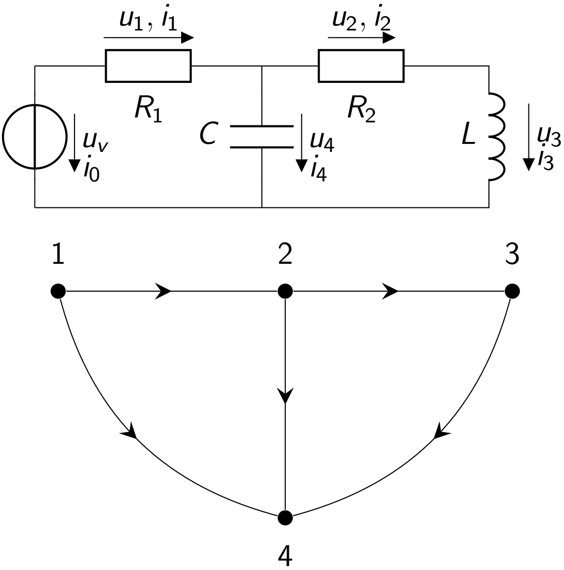
Inductors

- Types of inductors
  - general:
    $$\Psi(t) = \varphi(i(t))$$
  - linear:
    $$\Psi(t) = L(t) \cdot i(t)$$
    $L(t)$ is the *inductance*
  - linear time-invariant (constant inductance):
    $$L(t) = L$$
- LTI case:
  $$u(t) = L \cdot \frac{di(t)}{dt}$$

Basic principles: Kirchoff's laws

- The electrical network is a connection of two-poles
- It can be represented by a directed graph
  - vertices: connection points
  - edges: branches
  - direction of the edges = direction of the current flow
- Kirchoff's laws express the conservation of electrical charge and energy within electrical networks

Basic principles: Kirchoff's laws



Kirchoff's current law

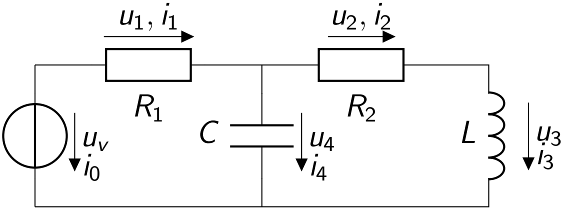
- The algebraic sum of currents in a network of two-poles meeting at a point is zero.

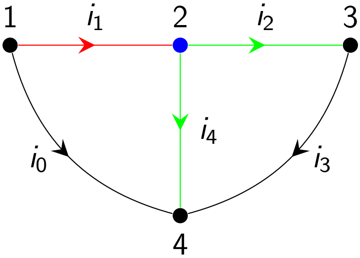
$$\sum_{k=1}^{K} i_k(t) = 0$$

in every time instant for a vertex joining K branches

- Input edges: $+$, Output edges: -
- Expresses the conservation of electrical charge
- Current $=$ time derivative of charge ($i(t) = \frac{dq(t)}{dt}$)
- Applies to every vertex of the graph

Kirchoff's current law





Example:
Kirchoff's current law for vertex 2:
$i_1 - i_2 - i_4 = 0$

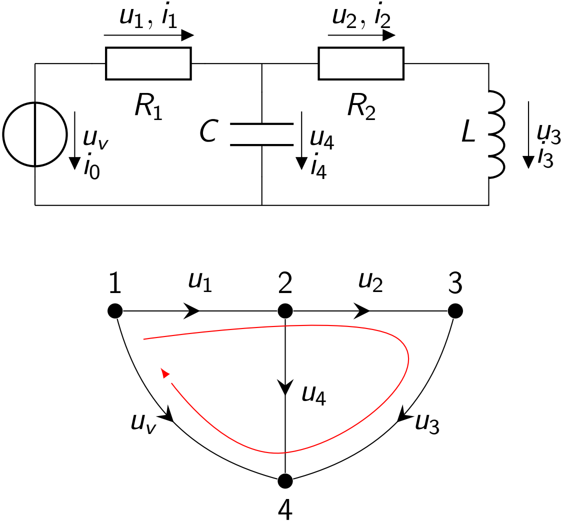
Kirchoff's voltage law

- The directed sum of the potential differences (voltages) around any closed loop is zero in every time instance.

$$\sum_k u_k(t) = 0$$

- Applies to every loop in the graph
- Sign of voltages:
  - If the edge and the path has the same direction: $+$
  - If the edge and the path has the opposite direction: -

Kirchoff's voltage law



Example:
Kirchoff's voltage law in the loop:

$$u_1 + u_2 + u_3 - u_v = 0$$

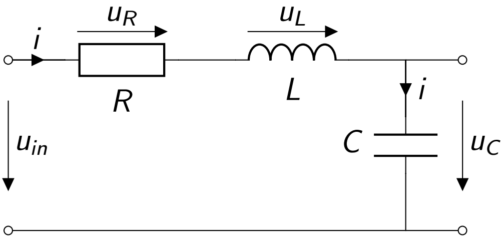
State space model

Systematic method to generate SS models:

1. Choose state variables: current of inductor, voltage of capacitor
2. Write model equations of capacitors and inductors in a state-variable format: derivative of the state variable on the left, everything else on the right
3. Find unknown variables in the state variable equations: only state variables and input variables are allowed
4. Use Kirchoff's laws and unused model equations to substitute and eliminate the unknown variables

Example: simple harmonic oscillator

- Components
  - voltage source u_{in}
  - resistor R
  - inductor L
  - capacitor C



Modelling assumptions

- Simple LTI two-poles are assumed.
- The power source u_{in} is the external input.

State space model

Model equations

- resistor: $u_R = R \cdot i$
- inductor: $u_L = L \cdot \frac{di}{dt}$
- capacitor $i = C \cdot \frac{du_c}{dt}$

Chosen state variables

- current: i
- voltage of the capacitor: u_c

Model equations in state-variable format

- $u_L = L \cdot \frac{di}{dt} \rightarrow \frac{di}{dt} = \frac{1}{L} \cdot u_L$
  $i = C \cdot \frac{du_C}{dt} \rightarrow \frac{du_C}{dt} = \frac{1}{C} \cdot i$

Unknown variables in the state equations: u_L

State space model

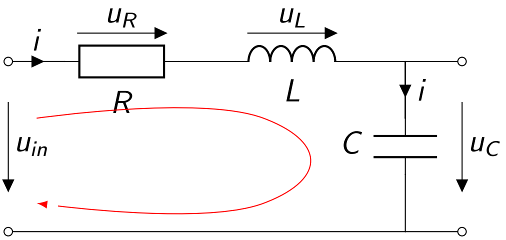
Eliminate u_L using Kirchoff's voltage law

$$u_R + u_L + u_C - u_{in} = 0$$

$$u_L = u_{in} - u_R - u_C = u_{in} - R \cdot i - u_C$$

substitute u_L into the equation of $\frac{di}{dt}$

$$\frac{di}{dt} = \frac{1}{L} \cdot (u_{in} - R \cdot i - u_C) = -\frac{R}{L} \cdot i - \frac{1}{L} \cdot u_C + \frac{1}{L} \cdot u_{in}$$



Building electrical models

- Aim: construct an equivalent circuit model that represents the behavior of the electrical system
  - e.g. a battery, PV panel, electric motor can be represented by an equivalent circuit
- How?
  - observe the nature of the phenomena, relationship of input-output data (eg. linear/nonlinear, exponential etc.)
  - analogies: electro-mechanical (e.g resistance $\approx$ damping), electro-thermal (e.g. voltage source $\approx$ temperature)
  - knowing the structure of the electrical system may help
  - the created model is not unique

Homework

Lowpass RLC filter

- Give the state space model of the electrical circuit, if the output is the u_R voltage