Computer Controlled Systems II.
Tutorial: Discrete diagnosis

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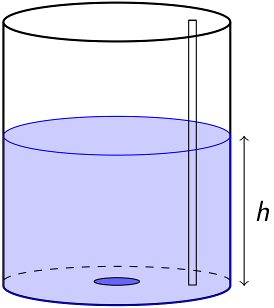
Tank with leak and additive sensor error

- Tank filled with water
- Normal operation
  - no inflow, no outflow
  - the water level is constant
- Level sensor
  - positive bias:
    the measured level is bigger than the real
  - negative bias:
    the measured level is bigger than the real
- Leak
  - gravitational outflow
  - small leak
  - large leak



Qualitative model

Qualitative range spaces

$$Q = \{0, L, N, H\}$$

$$Q_e = \{e^-, 0, L, N, H, e^+\}$$

$$X_l = \{0, 1\},\ X_s = \{-1, 0, 1\}$$

Qualitative difference equation of the tank

$$[h](k+1) = [h](k) - \chi_l [h][B_l]$$

$$[h] \in Q,\ \chi_l \in X_l$$

- small leak: $[B_l]$=L
- large leak: $[B_l]$=N

Qualitative algebraic equation of the sensor

$$[h^m](k) = [h](k) + \chi_s [B_s]$$

$$[h^m] \in Q_e,\ \chi_s \in X_s,\ [B_s] = L$$

Interval arithmetics

- addition
  - commutative (a+b=b+a)
  - identity element: 0 (a+0=0+a=a)
  - monotonic
- multiplication
  - commutative (a×b=b×a)
  - identity element: L (a×1=1×a=a)
  - zero element: 0 (a×0=0×a=0)
  - monotonic
- subtraction
  - not commutative
  - a-0=a
  - a-a=0
  - monotonic

+	0	L	N	H
0	0	L	N	H
L	L	N	H	e^+
N	N	H	e^+	e^+
H	H	e^+	e^+	e^+

×	0	L	N	H
0	0	0	0	0
L	0	L	N	H
N	0	N	H	H
H	0	H	H	H

-	0	L	N	H
0	0	e^-	e^-	e^-
L	L	0	e^-	e^-
N	N	L	0	e^-
H	H	N	L	0

Traces

	[h]	χ_l	χ_s	$[B_l]$	$[h^m]$
nominal	N	0	0	0	(N,N,N)
+bias	N	0	1	L	(H,H,H)
-bias	N	0	-1	L	(L,L,L)
leak	N	1	0	L	(0,0,0)
leak,+bias	N	1	1	L	(L,L,L)
leak,-bias	N	1	-1	L	(e^-,e^-,e^-)
leak	N	1	0	N	(0,0,0)
leak,+bias	N	1	1	N	(L,L,L)
leak,-bias	N	1	-1	N	(e^-,e^-,e^-)

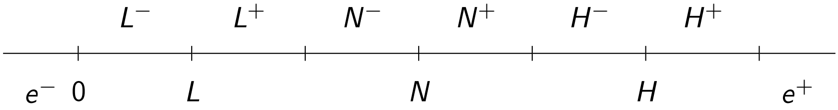
- only normal and +bias sensor error can be uniquely diagnosed

Refined qualitative range space

$$Q^* = \{0, L^-, L^+, N^-, N^+, H^-, H^+\}$$

$$[B_l] \in \{L^-, N^-\}$$

$$[B_s] = L^+$$



Extended arithmetic tables

Addition

$+$	0	L^-	L^+	N^-	N^+	H^-	H^+
0	0	L^-	L^+	N^-	N^+	H^-	H^+
L^-	L^-	L^+	N^-	N^+	H^-	H^+	e^+
L^+	L^+	N^-	N^+	H^-	H^+	e^+	e^+
N^-	N^-	N^+	H^-	H^+	e^+	e^+	e^+
N^+	N^+	H^-	H^+	e^+	e^+	e^+	e^+
H^-	H^-	H^+	e^+	e^+	e^+	e^+	e^+
H^+	H^+	e^+	e^+	e^+	e^+	e^+	e^+

Extended arithmetic tables

Multiplication (identity element: L^+)

$\times$	0	L^-	L^+	N^-	N^+	H^-	H^+
0	0	0	0	0	0	0	0
L^-	0	L^-	L^-	L^+	N^-	N^+	H^-
L^+	0	L^-	L^+	N^-	N^+	H^-	H^+
N^-	0	L^+	N^-	N^+	H^-	H^+	e^+
N^+	0	N^-	N^+	H^-	H^+	e^+	e^+
H^-	0	N^+	H^-	H^+	e^+	e^+	e^+
H^+	0	H^-	H^+	e^+	e^+	e^+	e^+

Extended arithmetic tables

Subtraction

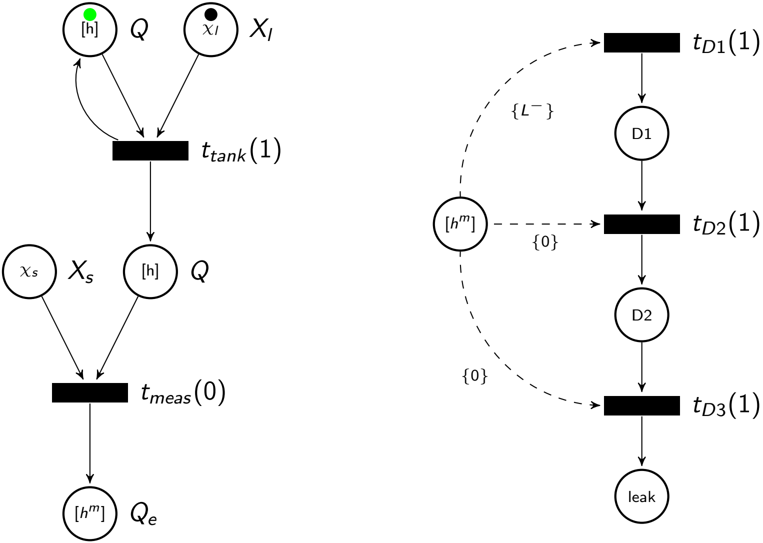
-	0	L^-	L^+	N^-	N^+	H^-	H^+
0	0	e^-	e^-	e^-	e^-	e^-	e^-
L^-	L^-	0	e^-	e^-	e^-	e^-	e^-
L^+	L^+	L^-	0	e^-	e^-	e^-	e^-
N^-	N^-	L^+	L^-	0	e^-	e^-	e^-
N^+	N^+	N^-	L^+	L^-	0	e^-	e^-
H^-	H^-	N^+	N^-	L^+	L^-	0	e^-
H^+	H^+	H^-	N^+	N^-	L^+	L^-	0

Traces with extended qualitative range spaces

	[h]	χ_l	χ_s	$[B_l]$	$[h^m]$
nominal	N^+	0	0	0	(N^+,N^+,N^+)
+bias	N^+	0	1	L^-	(H^+,H^+,H^+)
-bias	N^+	0	-1	L^-	(L^+,L^+,L^+)
leak	N^+	1	0	L^-	$(L^-,0,0)$
leak,+bias	N^+	1	1	L^-	(N^-, L^+, L^+)
leak,-bias	N^+	1	-1	L^-	(e^-,e^-,e^-)
leak	N^+	1	0	N^-	$(0,0,0)$
leak,+bias	N^+	1	1	N^-	(L^+, L^+, L^+)
leak,-bias	N^+	1	-1	N^-	(e^-,e^-,e^-)

- 5 operation mode can be uniquely diagnosed (nominal, +bias, small leak, small leak-bias, large leak)
- 4 faults cannot be diagnosed (-bias, large leak+bias, small leak-bias, large leak-bias)

Diagnoser



[h]
Q
χ_l
X_l
$t_{tank}(1)$
χ_s
X_s
[h]
Q
$t_{meas}(0)$
$[h^m]$
Q_e
$t_{D1}(1)$
$\{L^-\}$
D1
$[h^m]$
$t_{D2}(1)$
$\{0\}$
D2
$\{0\}$
$t_{D3}(1)$
leak