UIO for DC brushed motor

Let us have the LTI state-space model of a DC motor:

$$\begin{cases} \frac{\mathrm{d}i}{\mathrm{d}t} &= -\frac{R}{L}i - \frac{k_\omega}{L}\omega + \frac{1}{L}u \\ \frac{\mathrm{d}\omega}{\mathrm{d}t} &= \frac{k_i}{J}i - \frac{k_{f1}}{J}\omega - \frac{1}{J}\tau_{ext}, \text{ where} \\ \frac{\mathrm{d}\alpha}{\mathrm{d}t} &= \omega \end{cases}$$

$R = 7.13\Omega, L = 1.05 \cdot 10^{-3}H, k_\omega = 0.03823Vs/rad, k_i = 38.2 \cdot 10^{-3}Nm/A, J = 41.9 \cdot 10^{-7}kgm^2, k_f = 2.9264 \cdot 10^{-6}Nms/rad$ are the winding resistance, inductance, speed constant, torque constant, rotor inertia, Coulomb friction constant, respectively.

The state-space model looks like $\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) \\ y(t) &= Cx(t) + Du(t) \end{cases}$, with state vector $x = (i \quad \omega \quad \alpha)^T$. Here the external torque is considered a disturbance.

Problems:

1. <u>UIO design</u>: Design and simulate an observer for the above-defined system if the output matrices are defined as $C = I_3$, and $D = (0 \quad 0 \quad 0)^T$. Create a Simulink file for the system and the observer. Check for the observer rank condition. Calculate matrices H, T, K_1, K_2, F, K (check the detectability of the system, K_1 can be calculated with the help of *acker*, *place* or *lqr*). Simulate the system
2. <u>Fault detection</u>: Extend the current dynamics of the system with an induced constant current fault: $\frac{\mathrm{d}i}{\mathrm{d}t} = -\frac{R}{L}i - \frac{k_\omega}{L}\omega + \frac{1}{L}u + \frac{1}{L}f$, where $\frac{\mathrm{d}f}{\mathrm{d}t} = 0$. Design the UIO for the new system (using the steps from the previous problem), if the measurable outputs are the same as before. Is the observability rank condition fulfilled? Is the detectability condition met? What is the settling time of the observer (the time when $(f - \hat{f})/f \leq 2\%$)? What happens if we replace the constant *f* with a step function in the simulation?

- The answers should be submitted in a .m or preferably .mlx file accompanied by two Simulink (.slx) files (for problem 1 and 2).