INTELLIGENT CONTROL SYSTEMS

Qualitative modelling

Katalin Hangos

Department of Electrical Engineering and Information Systems

Oct 2020

Lecture overview

1. **Sign and interval calculus**
   - Sign addition and multiplication
   - Interval operations
2. **The notion of qualitative models**
3. **Signed Directed Graph (SDG) models**
   - Structure graph
   - Diagnostic reasoning
4. **Confluences**
   - Derivation and solution of confluences
   - Rule generation from confluences
5. **Qualitative difference equations**
   - The derivation and solution of QDEs
   - Rule generation from QDEs

Sign and interval calculus

Discrete range spaces

Universe: the range space of variables = a set of intervals

- *General qualitative*: real intervals with fixed or free endpoints

$$U_{\mathcal{I}} \;=\; \{[a_\ell, a_u] \mid a_\ell,\; a_u \in \mathcal{R},\; a_\ell \leq a_u\}$$

  with the **landmark set**

$$L_{\mathcal{I}} = \{a_i \mid a_i \leq a_{i+1}\;,\; i \in I \subseteq \mathcal{N}\}$$

- *sign-valued case*

$$\begin{aligned} U_{\mathcal{S}} &= \{\, + \,,\, - \,,\, 0 \,;\, ? \,\} \;,\; ? = + \cup 0 \cup - \\ L_{\mathcal{S}} &= \{\, a_1 = -\infty \,,\; a_2 = 0 \,,\; a_3 = \infty\} \end{aligned}$$

- *logical* (extended)

$$U_{\mathcal{L}} \;=\; \{\, \textbf{true} \,,\; \textbf{false} \,;\; \textbf{unknown} \,\}$$

Sign algebra

Algebra over the **sign universe**

Operations: with **usual algebraic properties** (commutativity, associativity, distributivity)

- sign addition ($\oplus_S$) and substraction ($\ominus_S$)
- sign multiplication ($\otimes_S$) and division
- composite operations and functions

Specification (definition) of a sign operation is done by using **operation tables**.

Recall - Logical operations

Operation table of the **implication** ($\rightarrow$) operation:

- used for describing **rules**

$a \rightarrow b$ $a \downarrow \quad b \rightarrow$	**false**	**true**
false	**true**	**true**
true	**false**	**true**

Sign addition

Operation table

$a \oplus_S b$	$+$	0	$-$	?
$+$	$+$	$+$	**?**	?
0	$+$	0	$-$	?
$-$	**?**	$-$	$-$	?
?	?	?	?	?

Properties:

- **Growing uncertainty**
- commutative

NOTES

Properties of sign addition: "inherited" from the properties of the "usual" addition

- commutativity : implies the symmetry of the operation table (around the main diagonal)
- null element (0): $a \oplus_S 0 = 0 \oplus_S a = a$
- operand monotonicity : if $a_1 \leq a_2$ then $(a_1 \oplus_S b) \leq (a_2 \oplus_S b)$, holds for both operands

Sign multiplication

Operation table

$a \otimes_S b$	$+$	0	$-$	?
$+$	$+$	0	$-$	?
0	0	0	0	**0**
$-$	$-$	0	$+$	?
?	?	**0**	?	?

Properties:

- **correcting** by zero values
- commutative

NOTES

Properties of sign multiplication: "inherited" from the properties of the "usual" multiplicaition

- commutativity : implies the symmetry of the operation table (around the main diagonal)
- null element (0): $a \otimes_S 0 = 0 \otimes_S a = 0$
- operand monotonicity : if $a_1 \leq a_2$ then $(a_1 \otimes_S b) \leq (a_2 \otimes_S b)$, holds for both operands
- *unit element* (+): $a \otimes_S + = + \otimes_S a = a$

Interval operations – 1

Operation on intervals with *fixed* endpoints

- **Set-type definition**: the sum (or product) of two intervals $\mathcal{I}_1 = [a_{1\ell}, a_{1u}]$ and $\mathcal{I}_2 = [a_{2\ell}, a_{2u}]$ from $U_{\mathcal{I}}$ is the smallest interval from $U_{\mathcal{I}}$ which covers the interval

$$\mathcal{I}^* = \{\ b = a_1 \ \mathbf{op}\ a_2 \mid a_1 \in \mathcal{I}_1\ ,\ a_2 \in \mathcal{I}_2\}$$

- **Endpoint-type definition**: for *monotonic operations* we can compute the above as

$$\begin{aligned} E_{op} = \{\ & e_{\ell\ell} = a_{1\ell}\ \mathbf{op}\ a_{2\ell}\ ,\ e_{\ell u} = a_{1\ell}\ \mathbf{op}\ a_{2u}\ , \\ & e_{u\ell} = a_{1u}\ \mathbf{op}\ a_{2\ell}\ ,\ e_{uu} = a_{1u}\ \mathbf{op}\ a_{2u}\ \} \\ & \text{with} \quad \mathcal{I}^* \ = \ [\ min\ E_{op},\ max\ E_{op}\ ] \end{aligned}$$

  where E_{op} is formed from the endpoints

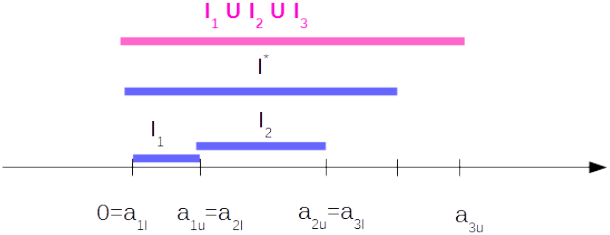
Interval operations – 2

Unusual properties caused by the fact that $\mathcal{I}^*$ *should be **covered*** by an interval from $U_{\mathcal{I}}$

- **growing uncertainty** with every operation
- **lack of distributivity**: the result may depend on the algebraic form
  minimum number of addition is the best

NOTES

The growing uncertainty is illustrated with the figure below.



Order of magnitude intervals

Universe

- landmark set:

$$L_{\mathcal{OM}} = \{\ a_1 = -\infty\ ,\ a_2 = -A\ ,\ a_3 = 0\ ,\ a_3 = A\ ,\ a_4 = \infty\}$$

- atomic intervals:

$$LN = [-\infty, -A), \quad SN = [-A, 0), \quad 0 = [0, 0], \quad SP = (0, A], \quad LP = (A, \infty]$$

$$U_{\mathcal{OM}} \ = \ \{\ LN\ ,\ SN\ ,\ 0\ ,\ SP\ ,\ LP\ \}$$

Non-atomic intervals and operations

- pseudo-intervals: $[SP, LP] = (0, \infty] \quad or \quad [LN, LP] = [-\infty, \infty]$
- operations: $LP\ \oplus_{OM}\ LN\ =\ [LN, LP]$

Order of magnitude addition

Operation table of the **order of magnitude** interval addition

$a \oplus_{OM} b$	LN	SN	0	SP	LP
LN	LN	LN	LN	$[LN, SN]$	$[LN, LP]$
SN	LN	$[LN, SN]$	SN	$[SN, SP]$	$[SP, LP]$
0	LN	SN	0	SP	LP
SP	$[LN, SN]$	$[SN, SP]$	SP	$[SP, LP]$	LP
LP	$[LN, LP]$	$[SP, LP]$	LP	LP	LP

NOTES

Properties of order of magnitude addition: "inherited" from the properties of the "usual" addition (see the *properties of sign addition*!)

- commutativity : implies the symmetry of the operation table (around the main diagonal)
- null element (0): $a \oplus_{OM} 0 = 0 \oplus_{OM} a = a$
- operand monotonicity : if $a_1 \leq a_2$ then $(a_1 \oplus_{OM} b) \leq (a_2 \oplus_{OM} b)$, holds fo both operands

Important

The growing uncertainty is seen from the non-atomic entries in the table

Normalized intervals

Qualitative range space: for variables with "normal" N value

$$\mathcal{Q} = \{H, N, L, 0\}, \quad \mathcal{B} = \{0, 1\}, \quad \mathcal{Q}_E = \{H, N, L, 0, e+, e-\}$$

Intervals with non-fixed endpoints to avoid growing uncertainty

Operation table for interval addition

$[a] \oplus_N [b]$	0	L	N	H
0	0	L	N	H
L	L	N	H	$e+$
N	N	H	$e+$	$e+$
H	H	$e+$	$e+$	$e+$

This is only a *possible definition* !

NOTES

Required properties **of normalized interval addition**:

- commutativity : implies the symmetry of the operation table (around the main diagonal)
- null element (0): $a \oplus_N 0 = 0 \oplus_N a = a$
- operand monotonicity : if $a_1 \leq a_2$ then $(a_1 \oplus_N b) \leq (a_2 \oplus_S b)$, holds for both operands

This allows some flexibility in the definition.
Another possible definition

$[a] \oplus_N [b]$	0	L	N	H
0	0	L	N	H
L	L	L	N	H
N	N	N	N	H
H	H	H	H	H

The notion of qualitative models

The notion of qualitative models

The range space of the variables and parameters is interval-valued

- *sign-valued*
  - Signed Directed Graph (SDG) models
  - Confluences (sign qualitative differential equations)
- *interval-valued*
  - Qualitative Differential Equations (QDEs): constraint type, algebraic type

From AI viewpoint: qualitative models are **special knowledge representation forms** with special reasoning.

The origin of qualitative models

Nonlinear dynamical models in **state-space form**:

$$\begin{aligned} \frac{dx}{dt} &= f(x,u) \quad \text{(state eq.)} \\ y &= h(x,u) \quad \text{(output eq.)} \end{aligned}$$

Qualitative models can be derived *systematically* from engineering models by using

- interval-values variables and parameters
- simplified equations

SDG models

The structure of a state-space model

Linearized state-space models near a *steady-state point*

$$\begin{array}{lcll} \frac{dx}{dt} & = & Ax + Bu & \text{(state eq.)} \\ y & = & Cx + Du & \text{(output eq.)} \end{array}$$

Signed structure matrices : $[A]$

$$[A]_{ij} = \begin{cases} + & \text{if} \quad a_{ij} > 0 \\ 0 & \text{if} \quad a_{ij} = 0 \\ - & \text{if} \quad a_{ij} < 0 \end{cases}$$

Structure graph

A signed directed graph $S = (V, \mathcal{E}; w)$

- **vertex set** for the state, input and output variables

$$V = X \cup U \cup Y$$
$$X \cap U = X \cap Y = U \cap Y = \emptyset$$

- **edges** for the *direct* effects between variables
- edge **weights** for the *sign* of the effect

Construction of the structure graph

Given a nonlinear state space model

$$\begin{aligned} \frac{dx}{dt} &= f(x,u) \quad \text{(state eq.)} \\ y &= h(x,u) \quad \text{(output eq.)} \end{aligned}$$

The structure graph $S = (V, \mathcal{E}; w)$ is constructed in three steps

1. **vertex set** for the state, input and output variables

$$V = X \cup U \cup Y$$
$$X \cap U = X \cap Y = U \cap Y = \emptyset$$

2. **edges** for the *direct* effects between variables
   - either from the linearized model equations
   - or by direct inspection of the model equations
3. edge **weights** for the *sign* of the effect

NOTES

Construction of the edges and edge weights by direct inspection of the model equations

- input $\rightarrow$ state edges : a directed edge $u_i \rightarrow x_j$ exists, if u_i is present in the right-hand side $f_j(x, u)$ of the jth state equation
- state $\rightarrow$ state edges : a directed edge $x_i \rightarrow x_j$ exists, if x_i is present in the right-hand side $f_j(x, u)$ of the jth state equation
- input $\rightarrow$ output edges : a directed edge $u_i \rightarrow y_j$ exists, if u_i is present in th right-hand side $h_j(x, u)$ of the jth output equation
- state $\rightarrow$ output edges : a directed edge $x_i \rightarrow y_j$ exists, if x_i is present in th right-hand side $h_j(x, u)$ of the jth output equation

Important

There are no edges directed to the input vertices.
There are no edges directed from the output vertices

Paths in the structure graph

A **directed path** $P = (v_1, v_2, ..., v_n)$, $v_i \in V$, $e_{i,i+1} = (v_i, v_{i+1}) \in \mathcal{E}$

- describe an indirect effect from variable v_1 to v_n
- **value** of the path

$$W(P) = \prod_{i=1}^{n-1} w(e_{i,i+1})$$

- significance of *shortest path(s)* and *directed circles*

Important

In the structure graph edges describe direct effects between variables, and directed paths correspond to indirect effects between them.

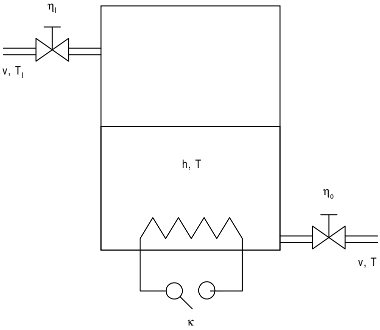
Diagnostic reasoning using SDGs

Sign-valued variables: sign of the *deviation from their steady-state value*

The effect of a variable v_i to another variable v_j

- **initial deviation** is determined by the **sign-value of the *shortest path(s)***
  - **sign-sum** is needed if not unique $\Longrightarrow$ **ambiguity**
- **steady-state effect** is the ***sign-sum* of the sign-value of *all* paths**
  - $\Longrightarrow$ **ambiguity** (often)
  - directed circles: solution of sign-linear equations

Example – Coffee machine



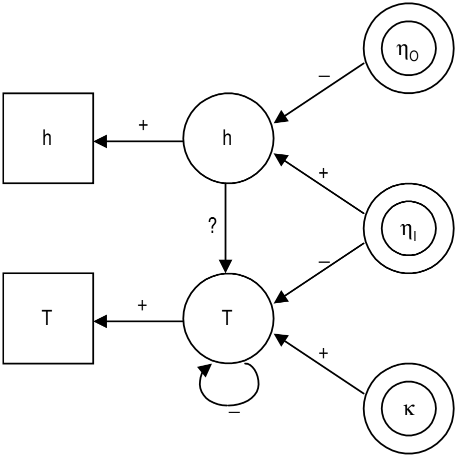
State-space model of the coffee machine

$$\frac{dh}{dt} = \frac{v}{A}\eta_I - \frac{v}{A}\eta_O \quad \text{(mass)}$$

$$\frac{dT}{dt} = \frac{v}{Ah}(T_I - T)\eta_I + \frac{H}{c_p \rho h}\kappa \quad \text{(energy)}$$

t	time $[s]$
h	level in the tank $[m]$
v	volumetric flowrate $[m^3/s]$
c_p	specific heat $[Joule/kgK]$
ρ	density $[kg/m^3]$
T	temperature in the tank $[K]$
T_I	inlet temperature $[K]$
H	heat provided by the heater $[Joule/sec]$
A	cross section of the tank $[m^2]$
η_I	binary input valve $[1/0]$
η_O	binary output valve $[1/0]$
κ	binary switch $[1/0]$

SDG of the coffee machine



Confluences

Origin of confluences

"Qualitative Physics" by de Kleer and Brown

Sign version of lumped nonlinear state equations (dynamic models with perfectly stirred balance volumes)

- can be *formally derived* therefrom
- *sign-valued variables and operations* are used

Important

A complete and contradiction-free rule-set can be derived *from confluences*

Derivation of confluences

1. **define qualitative variables $[q]$ and δq to each of the model variables $q(t)$ as follows**:

$$q \ \sim \ [q] = sign(q) \quad , \quad dq/dt \ \sim \ \delta q = sign(dq/dt)$$

2. **operations are replaced by sign operations**, i.e.

$$+ \ \sim \ \oplus_S \quad , \quad * \ \sim \ \otimes_S \quad etc.$$

3. **parameters are replaced by $+$ or $-$ or** 0 forming *sign constants* in the confluence equations, i.e. they virtually disappear from the equations.

Solution of a confluence

In the form of an **extended truth table**
(sign-operation table)

- collect all of the *right-hand side variables* (time-dependent values!)
- enumerate all of their sign-values
- systematically enumerate all of the **possible combinations**
  $\Longrightarrow$ exponentially growing size with the number of variables

Rule generation from confluences

The **rows of the truth table of a confluence can be interpreted as a rule** if one reads them from right to left.
For example

$$\delta h = [\eta_I] \ominus_S [\eta_O]$$

with the combination $\eta_I = 0$, $\eta_I = +$ gives $\delta h = -$

$\Longrightarrow$

$$\textbf{if} \;\; (\eta_I = \textbf{closed}) \textbf{ and } (\eta_O = \textbf{open}) \;\; \textbf{then} \;\; (h = \textbf{decreasing})$$

Important

Rule sets can be generated from the truth table of a confluence.
The generated rules are datalog rules.
The generated rule set is contradiction-free by construction ,
but it may not be complete.

NOTES

Important

*The **lack of completeness** of a generated rule set is the consequence of the fact, that one does not generate a rule of the consequence has the value ? (i.e., **unknown sign**).*

A simple example – 1

Model equation: mass balance of the coffee machine

$$\frac{dh}{dt} = \frac{v}{A}\eta_I - \frac{v}{A}\eta_O$$

1. **qualitative variables**: $[\eta_I] \in \{0, +\}$, $[\eta_O] \in \{0, +\}$
2. all **sign constants** are "$+$"
3. **confluence**

$$\delta h = [\eta_I] \ominus_S [\eta_O]$$

A simple example – 2

Truth table of the confluence

$$\delta h = [\eta_I] \ominus_S [\eta_O]$$

δh	$[\eta_I]$	$[\eta_O]$
0	0	0
$-$	0	$+$
$+$	$+$	0
?	$+$	$+$

Qualitative difference equations

The derivation of discrete time qualitative DAEs

Dynamic models derived from first engineering principles: continuous time differential-algebraic equation models

- differential equations originate from conservation balances: ***to be transformed to difference equations*** (time discretization)
- selection of the ***qualitative range spaces*** of variables and parameters
- deriving the qualitative form

Qualitative signals – 1

Qualitative range spaces

$$\mathcal{Q} = \{H, N, L, 0\}, \quad \mathcal{B} = \{0, 1\}, \quad \mathcal{Q}_{\mathcal{E}} = \{H, N, L, 0, e+, e-\}$$

with *H*igh, *L*ow, *N*ormal, *e*rror.

Important

A qualitative signal *is a signal (input, output, state and* ***disturbance (fault indicator)****) that takes its values from a finite qualitative range set*

An event is generated when a qualitative signal changes its value. An event e_X is formally described by a pair $e_X(t, q_X) = (t, [x](t) = q_X)$ where t is the occurrence time when the qualitative signal $[x]$ takes the value q_X.

Qualitative signals – 2

Signal trace : a sequence of events related to a qualitative signal $[x]$ with values in $q_X \in \mathcal{Q}_\mathcal{E}$

$$\mathcal{T}_{(x,k_1,k_N)} = \{[x](k_1), ..., [x](k_N)\} = \{q_{X1}(k_1), ..., q_{XN}(k_N)\}$$

Simplified notation: time is omitted

$$\mathcal{T}_x = (q_{X1}, ..., q_{XN})$$

e.g. with normalized intervals $\mathcal{Q}_E = \{H, N, L, 0, e+, e-\}$

$$(N, N, L, 0) \quad , \quad (N, N, N) \quad , etc.$$

Solution of a qualitative DAE

In the form of a **solution table**
(interval operation table)

- collect all of the *right-hand side variables* (time-dependent values!)
- enumerate all of their signal traces
- systematically enumerate all of the **possible combinations**
  $\Longrightarrow$ exponentially growing size with the number of variables

A static example: sensor with additive type fault

Algebraic model equation: $v^m = v + \chi \cdot E$
$[v] \in \mathcal{Q}$, $[v]^m \in \mathcal{Q}_e$, $\chi \in B_{-1} = \{-1, 0, 1\}$ and $[E] = L$

$[v^m]$	$[\chi]$	$[v]$	mode
N	0	N	normal
H	0	H	normal
L	0	L	normal
0	0	0	normal
$e+$	1	H	faulty
H	1	N	faulty
N	1	L	faulty
L	1	0	faulty
N	-1	H	faulty
L	-1	N	faulty
0	-1	L	faulty
$e-$	-1	0	faulty

The applied operation table for the normalized intervals should also be defined!

Rule generation from QDEs

The **rows of the solution table of a QDE can be interpreted as a rule** if one reads them from right to left.
For example

$$v^m = v + \chi \cdot E$$

with the combination $[v] = N$, $\chi = -1$ gives $[v]^m = L$
$\Longrightarrow$

$$\textbf{if} \;\; (\chi = \textbf{neg fault}) \textbf{ and } ([v] = \textbf{normal}) \;\; \textbf{then} \;\; ([v]^m = \textbf{low})$$

Important

Rule sets can be generated from the truth table of a static QDE in a datalog form.
The generated rules are contradiction-free and complete.

NOTES

Important

Completeness of the generated rule set follows from the operation table of normalized intervals.

Because of the non-fixed endpoints, the result of an algebraic manipulation with operand of atomic value has also an atomic value. This means that no **unknown** valued consequence exists (no growing uncertainty).

A dynamic example: mass balance of the coffee machine

Differential equation in discrete form: $h^{+1} = h + \chi_I \cdot v - \chi_O \cdot v$
$[h], [h]^{+1} \in \mathcal{Q}_e$, $\chi_I, \chi_O \in \mathcal{B}$ and $[v] = L$
Solution with constant inputs

$[h]^{+1}$	$[h](t_0)$	χ_I	χ_O
(N, N, N)	N	(1,1,1)	(1,1,1)
(L, L, L)	L	(1,1,1)	(1,1,1)
...	...	...	...
(N, N, N)	N	(0,0,0)	(0,0,0)
...	...	...	...
$(H, e+, e+)$	N	(1,1,1)	(0,0,0)
$(N, H, e+)$	L	(1,1,1)	(0,0,0)
...	...	...	...
$(L, 0, e-)$	N	(0,0,0)	(1,1,1)
$(0, e-, e-)$	L	(0,0,0)	(1,1,1)
...	...	...	...