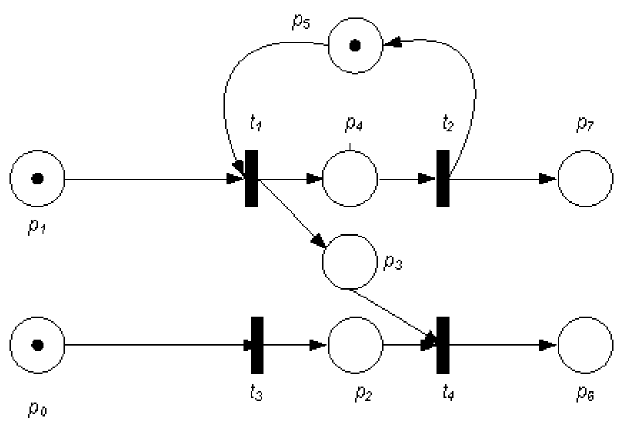
Discrete and Continuous Dynamical Systems – tutorial
Analysis of Petri net models

1 Analysis of Petri net models

Let us given the following Petri net with its graphical description



The following tasks should be carried out

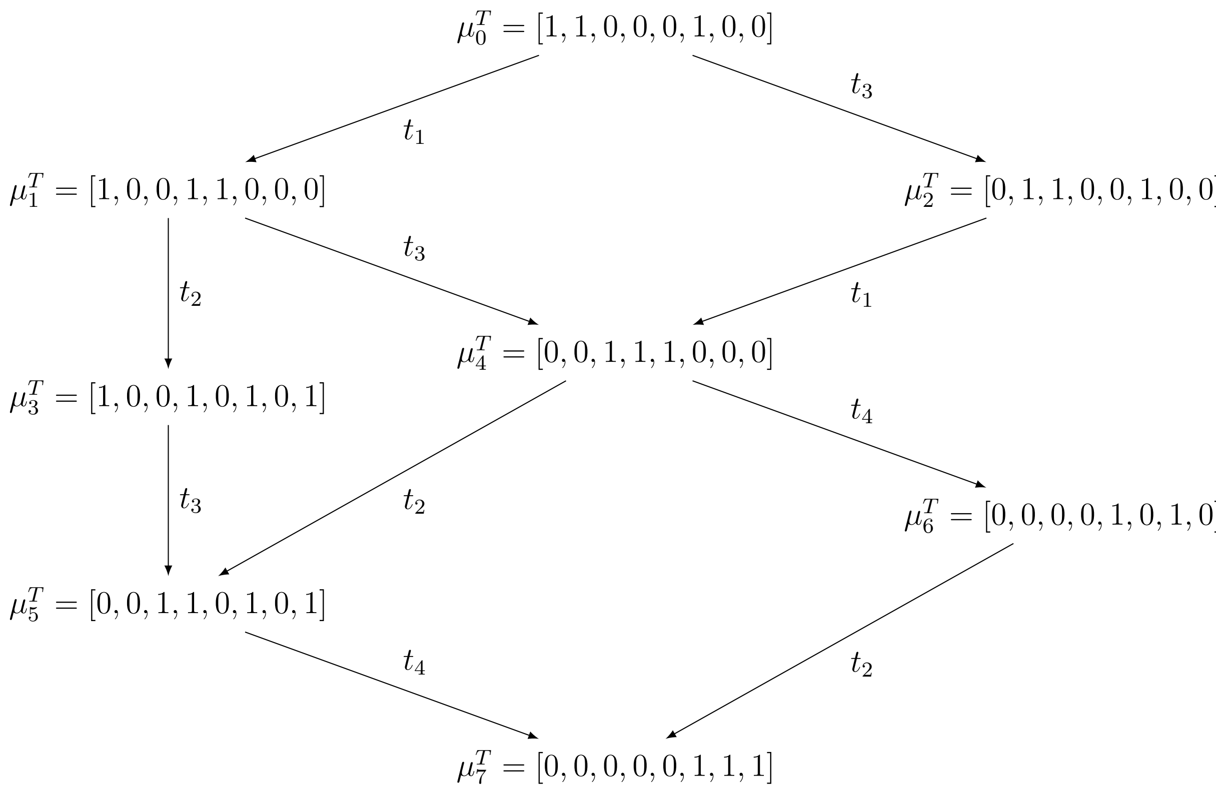
1. Construct the reachability graph from the marking vector μ_0 (shown in the figure) as an initial state.
2. Determine the input and output places from the net. Consider the set of final states those states that have a token on all of their output places.
3. Answer the following questions related to the behavioural properties of the Petri net:

- Are the final states reachable from the initial state?
- Is there any deadlock? If yes, give the deadlock.
- Is the Petri net bounded/safe? If yes, give the bound, if not give the non-bounded place.

1.1 Solution

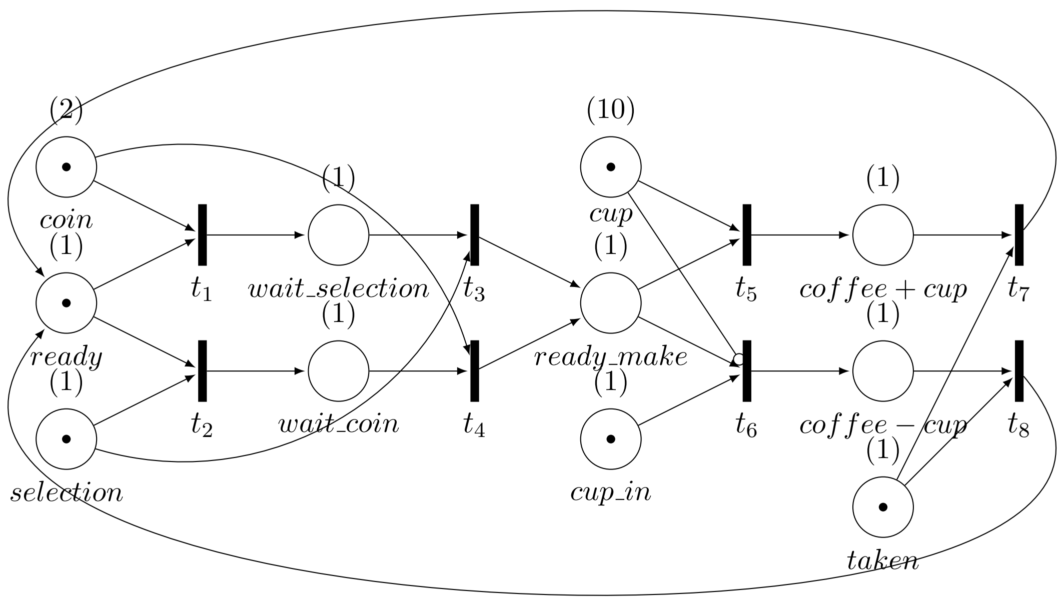
1. Construct the reachability graph from the marking vector μ_0 (shown in the figure) as an initial state.

    - At the initial state two transitions are enabled: t_1 and t_3. Draw two arcs from μ_0 and label them with t_1 and t_3.
    - By firing t_1 we get $\mu_1^T = [1, 0, 0, 1, 1, 0, 0, 0]$. At this state t_2 and t_3 are enabled. Draw two arcs fro μ_1 and label them with t_2 and t_3.
    - Return to μ_0 and fire t_3. We get $\mu_2^T = [0, 1, 1, 0, 0, 1, 0, 0]$. At this state only t_1 is enabled.
    - Return to μ_1 and fire t_2. We get $\mu_3^T = [1, 0, 0, 1, 0, 1, 0, 1]$. At this state only t_3 is enabled.
    - Return to μ_1 and fire t_3. We get $\mu_4^T = [0, 0, 1, 1, 1, 0, 0, 0]]$. At this state t_2 and t_4 are enabled.
    - Return to μ_2 and fire t_1. We get the same state as μ_4, therefore no new node has to be added to the graph. Draw an arc from μ_2 to μ_4 and label it with t_1.
    - Continue with μ_3. Fire the only enabled transition t_3. We get $\mu_5^T = [0, 0, 1, 1, 0, 1, 0, 1]$. At this state only t_4 is enabled.
    - Return to μ_4 and fire t_2. We get the same marking as μ_5 therefore we draw an arc to μ_5 and label it with t_2.
    - At μ_4 fire the other transition which is t_4. We get a new marking $\mu_6^T = [0, 0, 0, 0, 1, 0, 1, 0]$. At this state only t_2 is enabled.
    - Return to μ_5 and fire t_4. We get a new marking $\mu_7^T = [0, 0, 0, 0, 0, 1, 1, 1]$. This is a terminal node because there is no enabled transition at this state.
    - Return to μ_6 and fire t_2. We get the same marking as μ_7 therefore we draw an arc to μ_7.
    - The construction of the reachability graph is finished because there is no unprocessed node in the graph. The full reachability graph can be seen in the figure below.



2. Determine the input and output places from the net. Consider the set of final states those states that have a token on all of their output places.

   - The input places are the places with no input arc. In this example the input places are p_0 and p_1.
   - The output places are the places with no output arc. In this example the output places are p_6 and p_7.
   - The internal states are the places with both input and output arcs. In this example the internal places are p_2, p_3, p_4 and p_5.
   - There is only one final state which is $\mu_7^T = [0,0,0,0,0,1,1,1]$.
   - Answer the following questions related to the behavioural properties of the Petri net:
     - Are the final states reachable from the initial state? - Yes, reachable. See the reachability graph.



- Is there any deadlock? If yes, give the deadlock. - No, there is no deadlock.
- Is the Petri net bounded/safe? If yes, give the bound, if not give the non-bounded place. - The Petri net is bounded and safe, because the maximum number of tokens on each place is 1.

2 Analysis of the Petri net model of the coffee making automaton

Consider the Petri net model of the simple coffee making automaton developed before with its meaningful initial and final states.

Check the properties of the model:

1. boundedness, conservation;
2. reachability of the final states,
3. possible deadlock states.

2.1 Solution

Consider the Petri net model of the simple coffee making automaton developed before with its meaningful initial and final states.

- Let the initial state is the one depicted in the figure: places *coin_in*, *m_ready*, *selection*, *cups*, *cup_in* and *taken* has one token.
- Let the final state is when there is no token on place *coin_in*, *selection*, *taken* and *cups* and there is one token on place *m_ready*.

Check the properties of the model:

1. boundedness, conservation;
    - The Petri net is bounded, because each place has a capacity.
    - The Petri net is not conservative, because the number of tokens is not constant. For example at the first step of the simulation (firing t_1 or t_2) the number of tokens is decreased by 1.
2. reachability of the final states,
    - The final state is reachable from the given initial state. However there may be initial states from which the final state is not reachable, for example: there is no cups in the machine and you do not have your own cup.
3. possible deadlock states.
    - Considering the given initial and final states, there is no deadlock in the model.

3 Homework:

(a) Consider the graphical description of your Petri net given by the png file named after your Neptun ID.

1. Construct the reachability graph from the marking vector μ_0 as an initial state.
2. Determine the input and output places from the net. Consider the set of final states those states that have a token on all of their output places.
3. Answer the following questions related to the behavioural properties of the Petri net:
   - Are the final states reachable from the initial state?
   - Is there any deadlock? If yes, give the deadlock.
   - Is the Petri net bounded/safe? If yes, give the bound, if not give the non-bounded place.

(*) **(Supplementary)**
Consider your Petri net that describes the action sequence which is necessary to operate a lift in a two-storied building with its initial states and possible final states. Check the properties of the model:

1. boundedness, conservation;
2. reachability of the final states,
3. possible deadlock states.

Deadline of submission: 2021.05.08. 10am