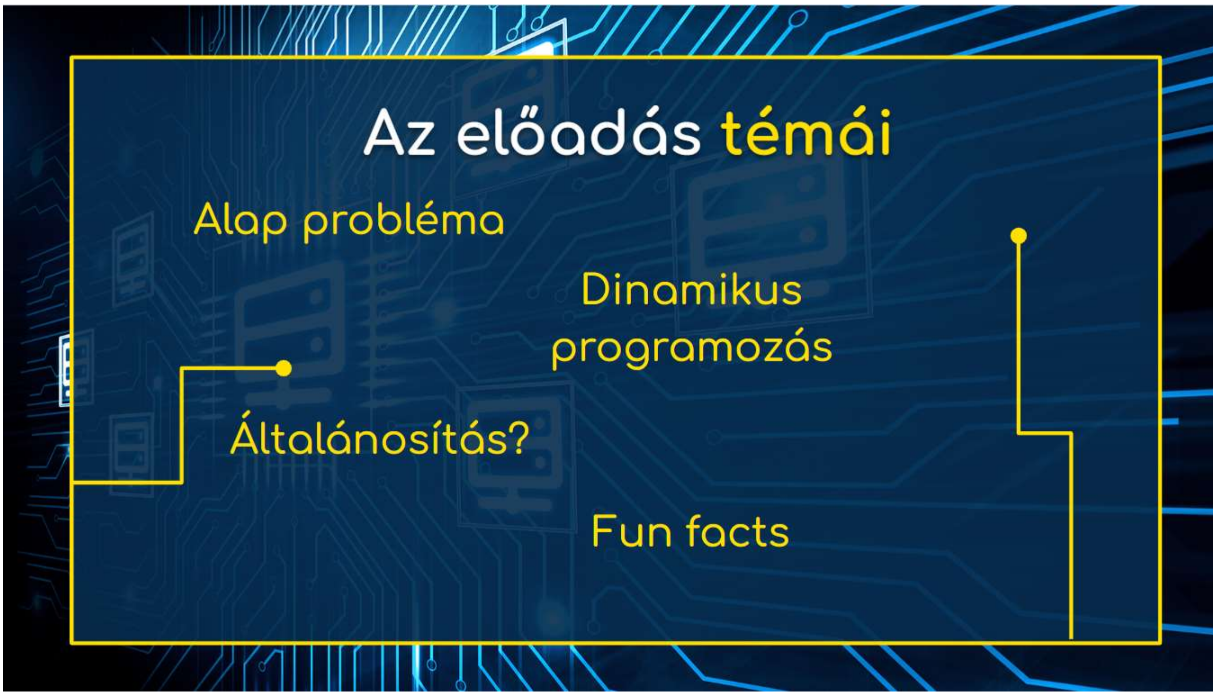
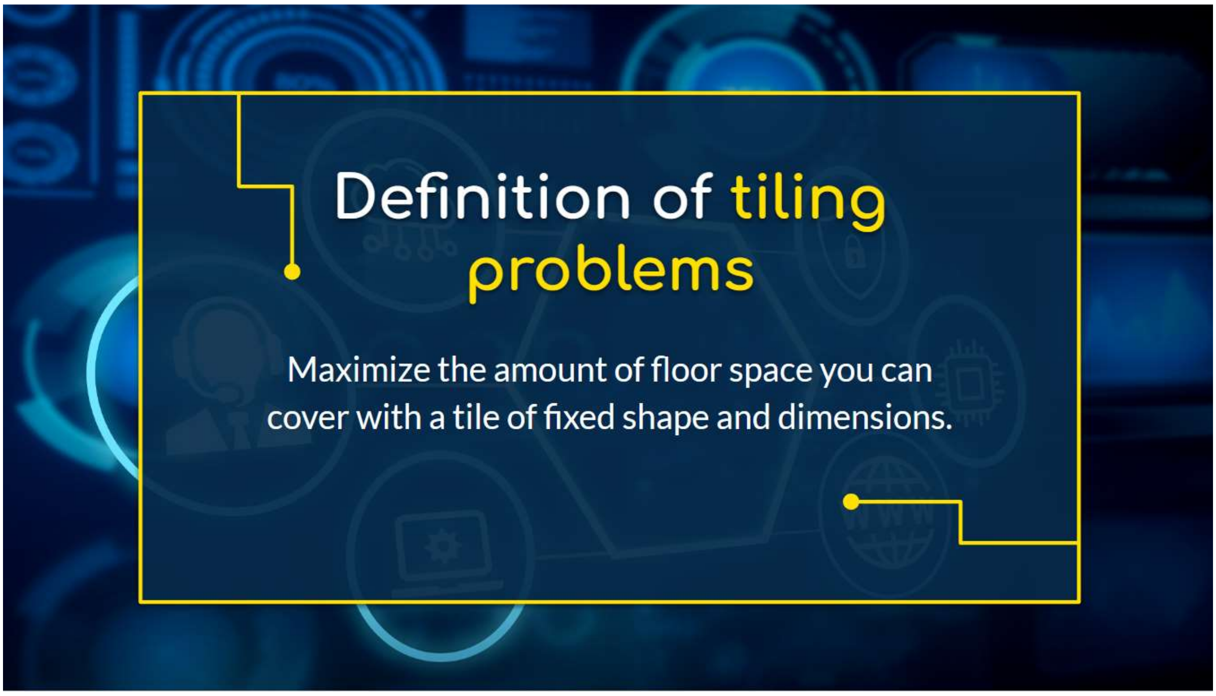
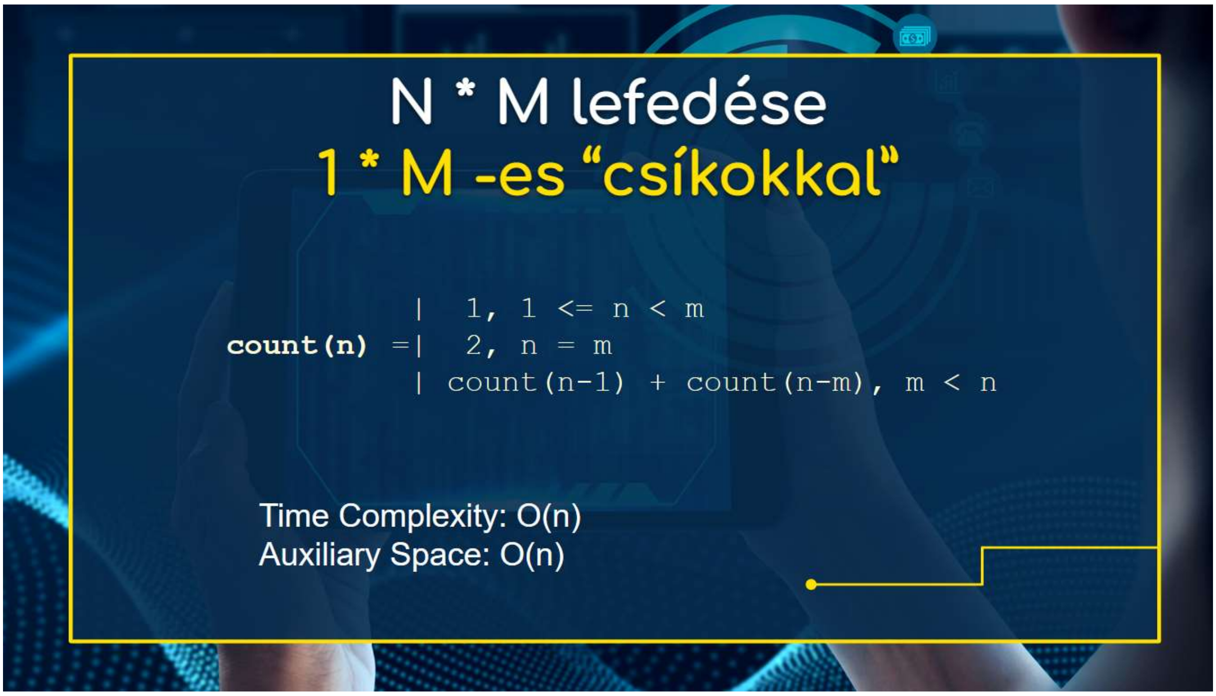

Tiling Problems
Haladó Informatikai Algoritmusok
3. Előadás - Princzes Barnabás

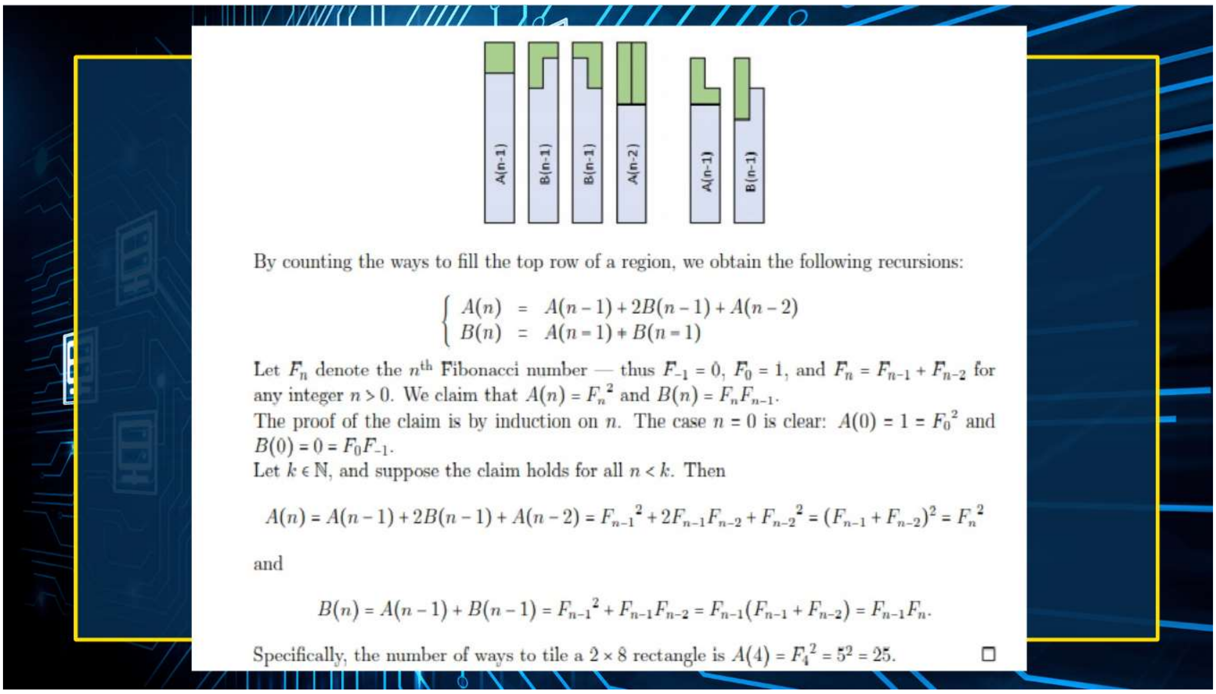

Az előadás témái
Alap probléma
Dinamikus programozás
Általánosítás?
Fun facts



https://en.wikipedia.org/wiki/The_Man_Who_Loved_Only_Numbers



https://www.geeksforgeeks.org/count-number-ways-tile-floor-size-n-x-m-using-1-x-m-size-tiles/
1 -> vonal vagy függőleges csíkozás



By counting the ways to fill the top row of a region, we obtain the following recursions:

$$\begin{cases} A(n) &= A(n-1) + 2B(n-1) + A(n-2) \\ B(n) &= A(n-1) + B(n-1) \end{cases}$$

Let F_n denote the n^{th} Fibonacci number — thus $F_{-1} = 0$, $F_0 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for any integer $n > 0$. We claim that $A(n) = F_n{}^2$ and $B(n) = F_n F_{n-1}$.
The proof of the claim is by induction on n. The case $n = 0$ is clear: $A(0) = 1 = F_0{}^2$ and $B(0) = 0 = F_0 F_{-1}$.
Let $k \in \mathbb{N}$, and suppose the claim holds for all $n < k$. Then

$$A(n) = A(n-1) + 2B(n-1) + A(n-2) = F_{n-1}{}^2 + 2F_{n-1}F_{n-2} + F_{n-2}{}^2 = (F_{n-1} + F_{n-2})^2 = F_n{}^2$$

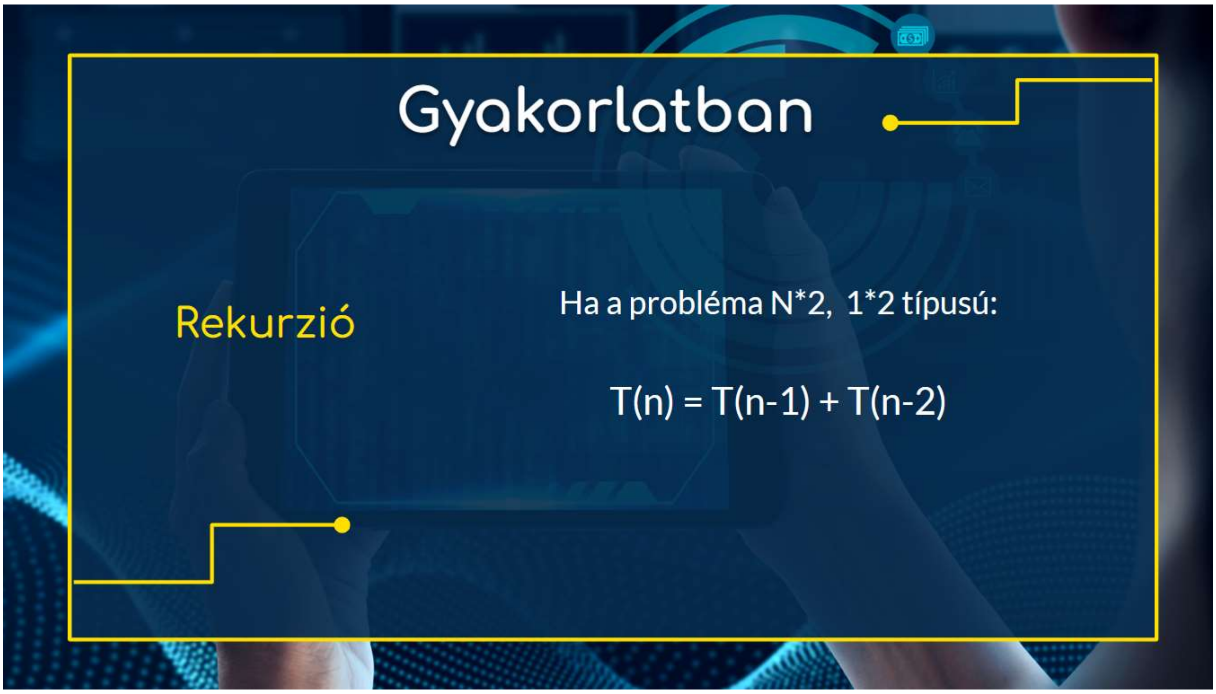
and

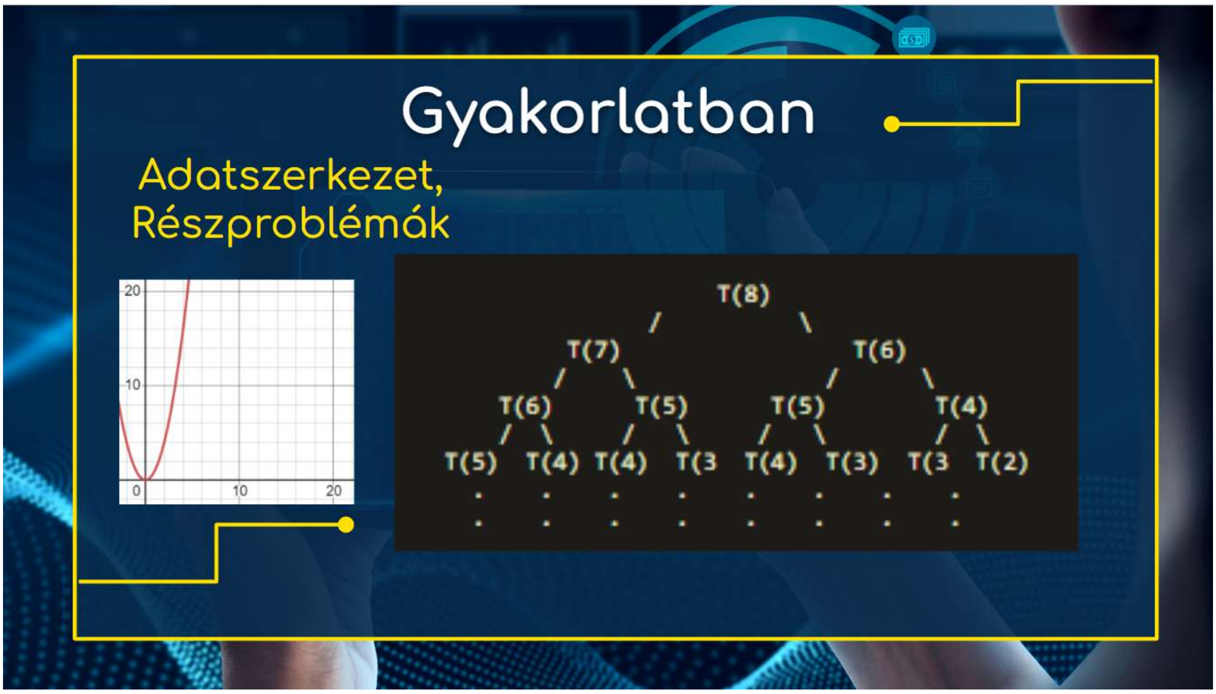
$$B(n) = A(n-1) + B(n-1) = F_{n-1}{}^2 + F_{n-1}F_{n-2} = F_{n-1}(F_{n-1} + F_{n-2}) = F_{n-1}F_n.$$

Specifically, the number of ways to tile a 2×8 rectangle is $A(4) = F_4{}^2 = 5^2 = 25$. $\square$

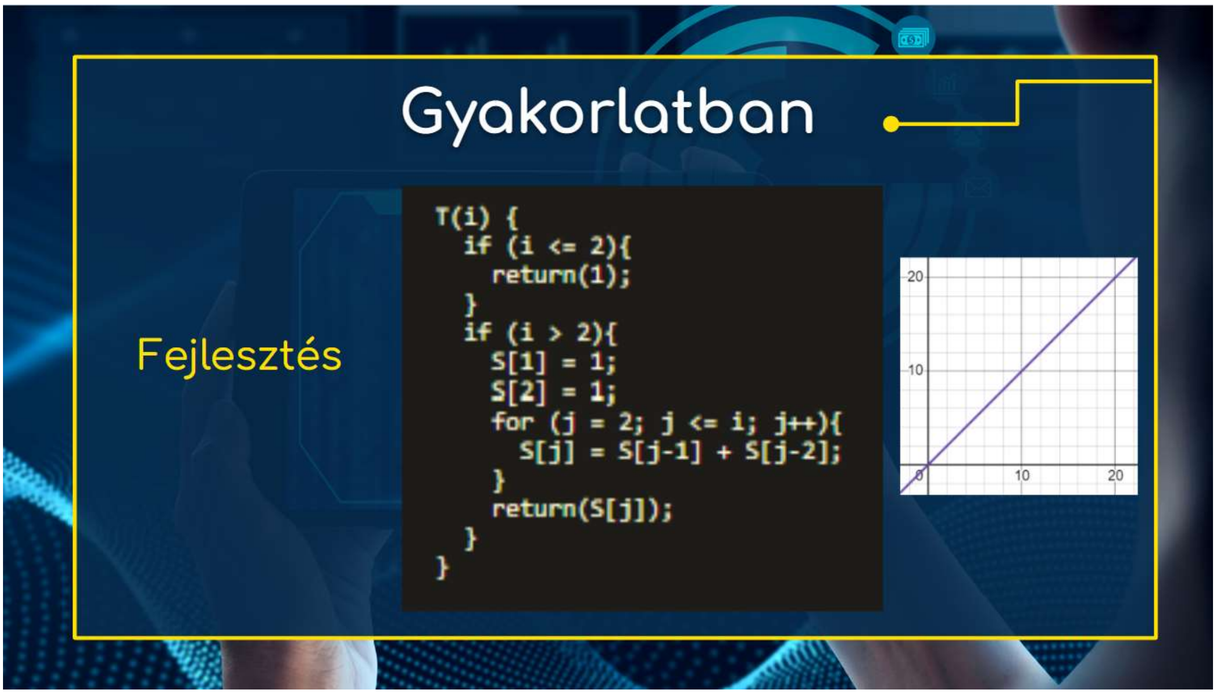
https://math.stackexchange.com/questions/3025592/tetris-tiling-2xn-or-filling-a-tube

Szegedi tudomány egyetem alapján
https://www.inf.szte.hu/~rfarkas/Alga17/alga-gyak-04.pdf

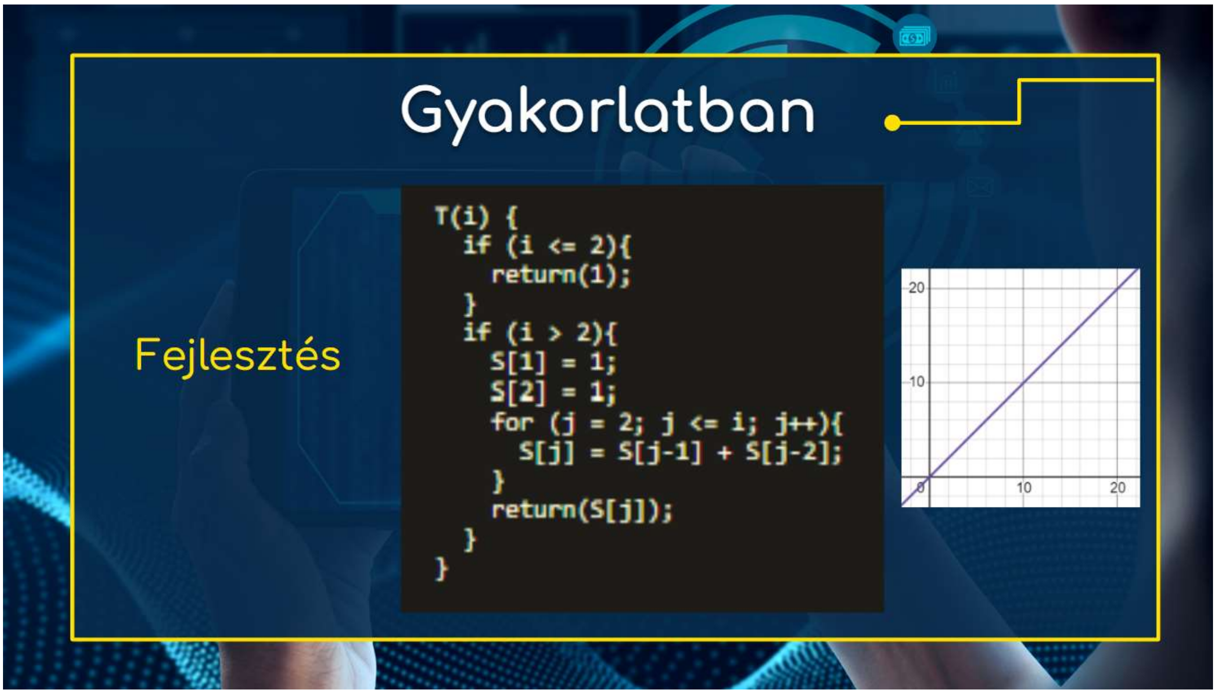




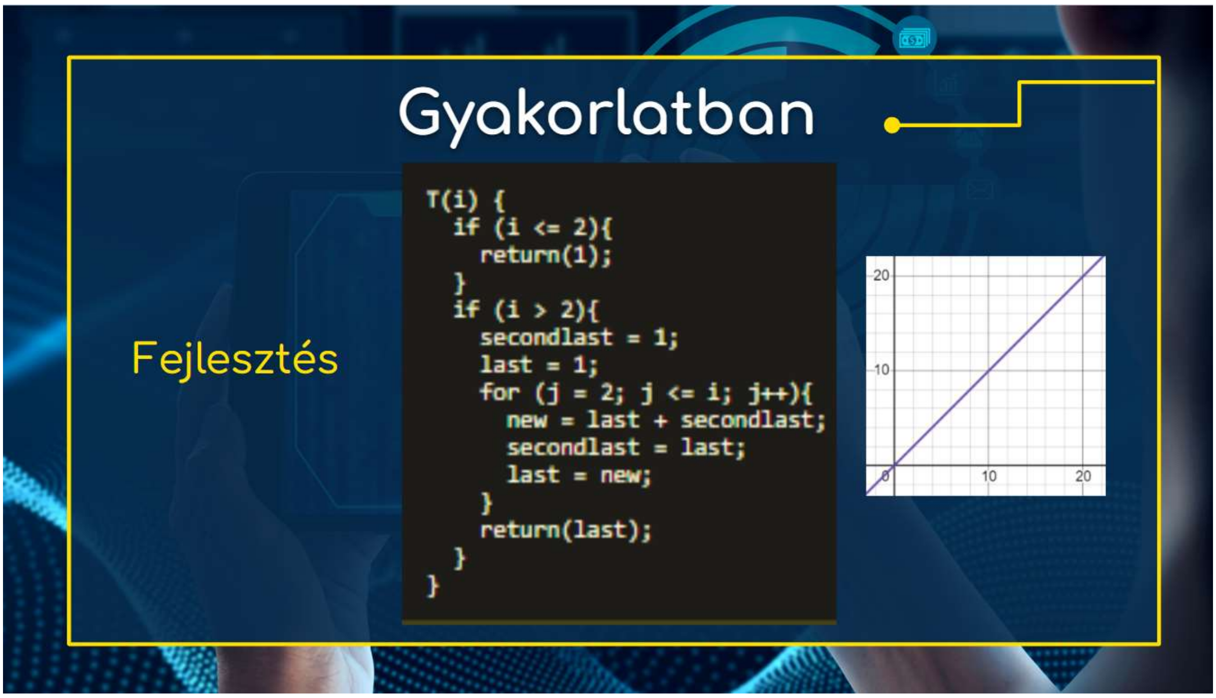
T(6) meg ilyenek többször ki vannak számolva
Fibonacci O(n^2)



Tárolhatjuk, de jobb ha inkább felfele megyünk



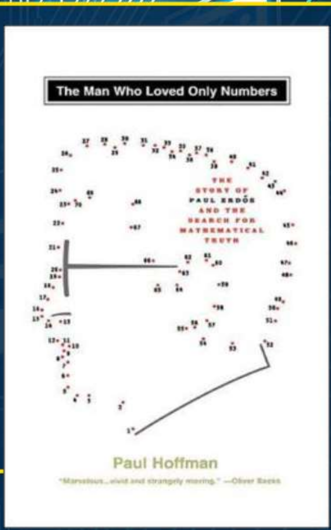
Mi lineáris még? Tárterület szükségessége



Tárolhatjuk, de jobb ha inkább felfele megyünk
L alakra is megmutatják

2*1-es dominók
https://en.wikipedia.org/wiki/Mutilated_chessboard_problem

Definíció



The Man Who Loved Only Numbers - Paul Hoffman

Fun Facts

Magyarázatok, problémák

Project euler -> 114 - 117

https://www.youtube.com/@WilliamFiset-videos